Propagation of Exponential Magneto Radiative Shock Waves

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Abstract. In this present paper the self similar model of exponential spherical, cylindrical and plane shock wave is studied, taking magnetic radiative heat flux into account where total energy of the wave is variable and atmosphere is uniform.
Keywords. Spherical, Cylindrical, Radiative, Exponential, Similarity variables.
Mathematical Subjects Classification (2000): 76L05

1. INTRODUCTION

Wang [1], Koch [2], Helliwell [3], Ray and Banerjee [4] and others have investigated the propagation of plane shock waves in optically thick and thin limit cases of gas in detail. Gusev. [5], Ranga Rao and Ramana [6] have studied the problem of unsteady self similar motion of a gas displaced by a piston, according to an exponential law. Verma and Singh [7] and Singh and Srivastava [8] have considered the problems of spherical shock waves in an exponentially increasing medium under the law uniform pressure, Srivastava [9] has studied the problem of magnetoradiative shock.

In the present paper we discussed the strong exponential spherical, cylindrical and plane shock waves in a uniform atmosphere with magnetic radiative effects the similarity solution have been developed when radiation heat is more important than the radiation pressure and radiation energy and opaque the shock to be transparent and isothermal. The total energy of waves as cube of shock radius.

2. EQUATION OF MOTION GOVERNING FLOW

The equation of flow behind a spherical, cylindrical and plane shock wave where j =0, 1,2 corresponding to plane, cylindrical & spherical and v = 0 for plane and v =1 for cylindrical , spherical both

$$\frac{d\rho}{dt} + \rho\frac{\partial u}{\partial r} + \frac{j\rho u}{r} = 0 , \tag{2.1}$$

$$\frac{du}{dt} + \frac{1}{\rho}\frac{\partial P}{\partial t} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{\upsilon h^2}{\rho r} = 0 , \tag{2.2}$$

$$\frac{dh}{dt} + \frac{h\partial u}{\partial r} + \frac{\upsilon hu}{r} = 0 , \tag{2.3}$$

$$\frac{dE}{dt} + P\frac{d}{dt}\left(\frac{1}{\rho}\right) + \frac{1}{\rho r^j}\frac{\partial}{\partial R}\left(Fr^j\right) = 0 , \tag{2.4}$$

$$E = \frac{p}{(\gamma - 1)\rho} , \tag{2.5}$$

$$p = \tau\rho T , \tag{2.6}$$

where u, ρ, p T, E, F and h are the velocity, density, pressure, temperature, energy, magnetic field, heat flux.

Assuming local thermodynamics equilibrium and taking Rosselland's diffusion approximation.

$$F = -\frac{c\mu}{3}\frac{\partial}{\partial r}(\sigma T^4) \tag{2.7}$$

where c the velocity of light, μ the mean flow path of radiation is a function of density and temperature. Following wang (1966), we take

$$\mu = \mu_0\, \rho^{\alpha} T^{\beta} , \tag{2.8}$$

μ_0 α,β are being constant.

The inner expanding surface moves with time according to all exponential law.

$$\bar{r} = A \exp (mt) \qquad (m > 0) \tag{2.9}$$

and since we have assumed self similarity. The shock will also move with time according to an exponential constants.

3. BOUNDARY CONDITION

The disturbance is headed by an isothermal shock, therefore the boundary condition are

$$u_1 = \left[1 - \frac{1}{\gamma M^2}\right] v , \qquad (3.\ )$$

$$\rho_1 = \gamma M^2 \rho_0 , \tag{3.2}$$

$$\rho_1 = \rho_0 v^2 , \tag{3.3}$$

$$F_1 = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right]\rho_0 v^3 , \tag{3.4}$$

$$h_1 = \gamma M^2 h_0 , \tag{3.5}$$

where subscripts 0 and 1 denote the regions immediately ahead and behind the shock front, respectively and v is the shock velocity,M denotes the mach number.

4. SIMILARITY SOLUTION

The similarity transformation for the problem under consideration are

$$\eta = \frac{r}{B \exp(mt)}, \tag{4.1}$$

$$u = vV(\eta) , \tag{4.2}$$

$$\rho = \rho_0 G(\eta) , \tag{4.3}$$

$$p = \rho_0 v p(\eta), \tag{4.4}$$

$$F = \rho_0 v^3 Q(\eta), \tag{4.5}$$

$$h = \sqrt{\rho_0} v H(\eta); \tag{4.6}$$

the variable η assumes the value 1 at the shock and $\bar{\eta}$ on the inner expanding surface. This enables us to express the radius of the inner expanding surface.

$$\bar{r} = \bar{\eta} R. \tag{4.7}$$

Now using the equation (2.6), (2.8) and (4.3)-(4.5), (4.7) into the equation (2.7)

$$Q = -NG^{\alpha-\beta-4} P^{\beta+4} \left[\frac{P'}{P} - \frac{G'}{G}\right] ; \tag{4.8}$$

with $\beta = -2, \alpha$ remaining arbitrary $(0, \leq \alpha \leq 2)$ and

$$N = \frac{4mc\, \mu_0 \sigma \rho_0^{\alpha-1}}{3T^{1(\beta-4)}} = \text{a dimensions radiation parameters .} \tag{4.9}$$

Making use of the equation (4.1) - (4.6), the equation (2.1) – (2.5) are transformed into

$$G' = \frac{G(\eta V' + jV)}{\eta(\eta - V)} , \tag{4.10}$$

$$P' = [HH'\eta - vH^2] - G[V'(\eta - V) - V] , \tag{4.11}$$

$$H' = (1-V)\left[HV' + \frac{\upsilon VH}{\eta}\right] , \tag{4.12}$$

$$Q' = \frac{P}{(\gamma-1)} + \frac{PG'}{G\eta}\frac{(2+V-\gamma)}{(\gamma-1)} - \frac{Qj}{\eta} - \frac{VP'}{\eta(\gamma-1)} , \tag{4.13}$$

$$V' = \frac{\frac{QG^{1-\alpha}(\eta-V)}{NP} - PjV + (\eta-V)\,[HH'\eta - vH^2 - G]}{[G(\eta-V)^2 - P]} \tag{4.14}$$

Where primes denotes differentiation with respect to η. The appropriate transformed shock conditions are.

$$V(1) = \left[1 - \frac{1}{\gamma M^2}\right], \tag{4.15}$$

$$G_{(1)} = \gamma M^2 , \tag{4.16}$$

$$P_{(1)} = 1 , \tag{4.17}$$

$$Q(1) = \frac{1}{2}\left[\frac{1}{\gamma^2 M^4} - 1\right], \tag{4.18}$$

$$H_{(1)} = \gamma M . \tag{4.19}$$

5. NUMERICAL RESULT

For exhibiting the numerical solutions, it is convenient to write the flow variables in the non dimensional forms as

$$\frac{u}{u_1} = \frac{V}{V_{(1)}}, \qquad \frac{\rho}{\rho_1} = \frac{G}{G_{(1)}},$$

$$\frac{p}{p_1} = \frac{P}{P_{(1)}}, \qquad \frac{F}{F_1} = \frac{Q}{Q_{(1)}},$$

$$\frac{h}{h_1} = \frac{H}{H_{(1)}};$$

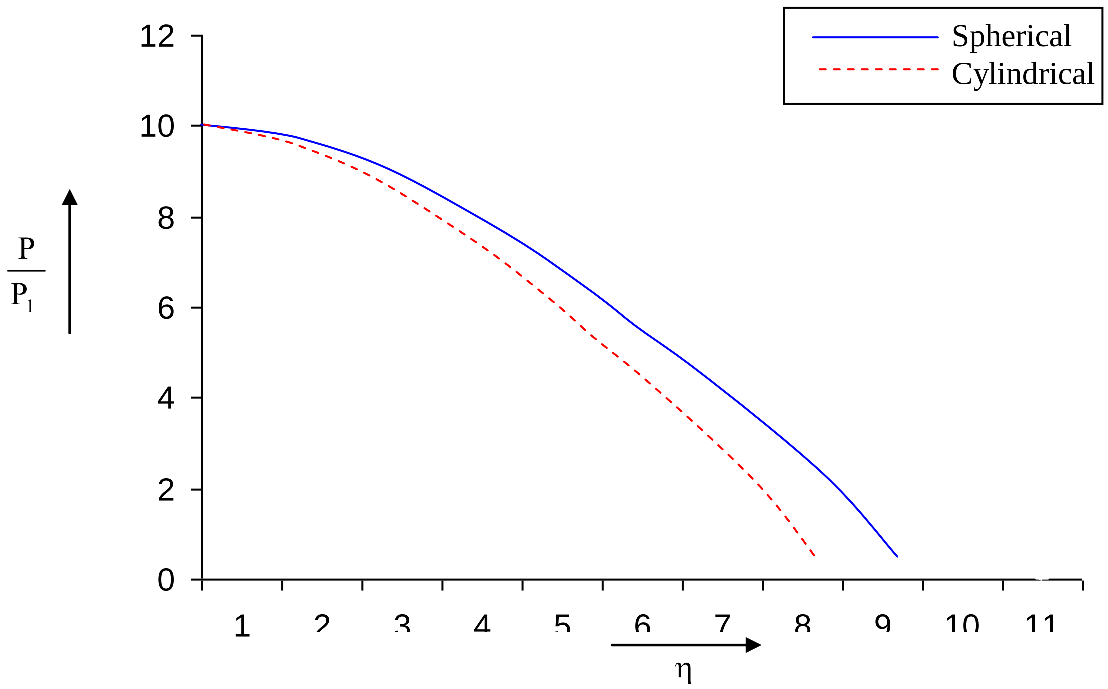
6. CONCLUSION

The numerical integration was carried out through using software matlab, for certain choice of parameter and reproduced in graphical form and nature of field variables is illustrated through them. We have calculated our result for following data.

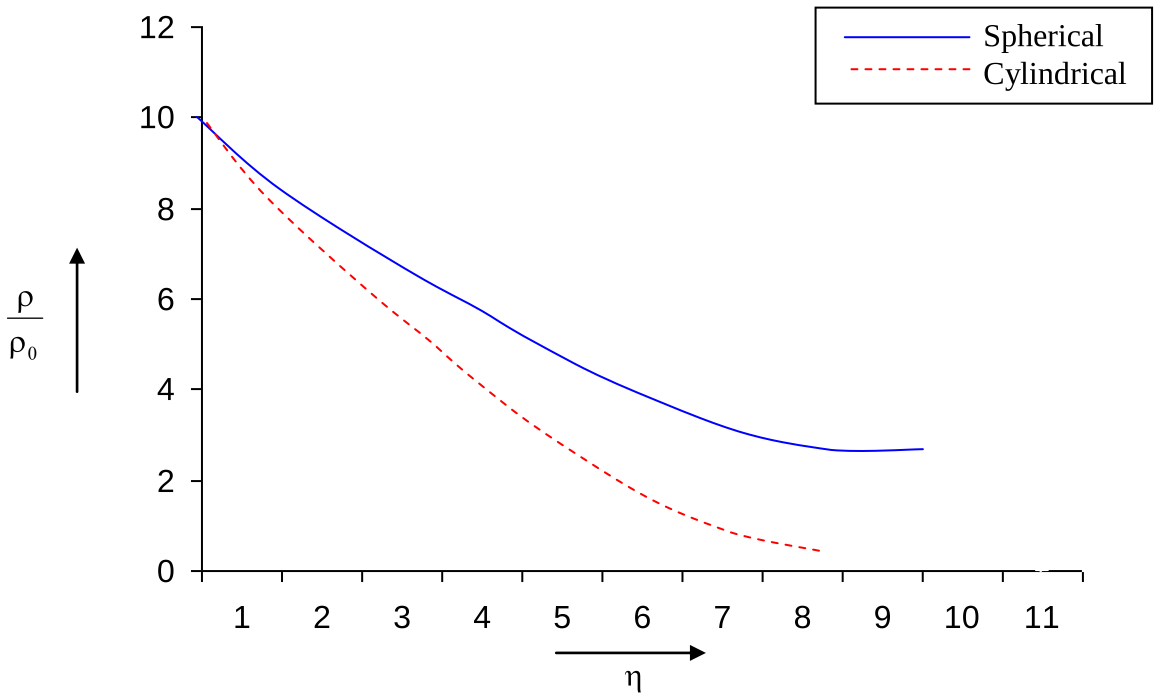
$\gamma = 1.4$, $M^2 = 1.01$, $\alpha = 0.25$.

At the shock surface pressure, density and magnetic field are maximum &

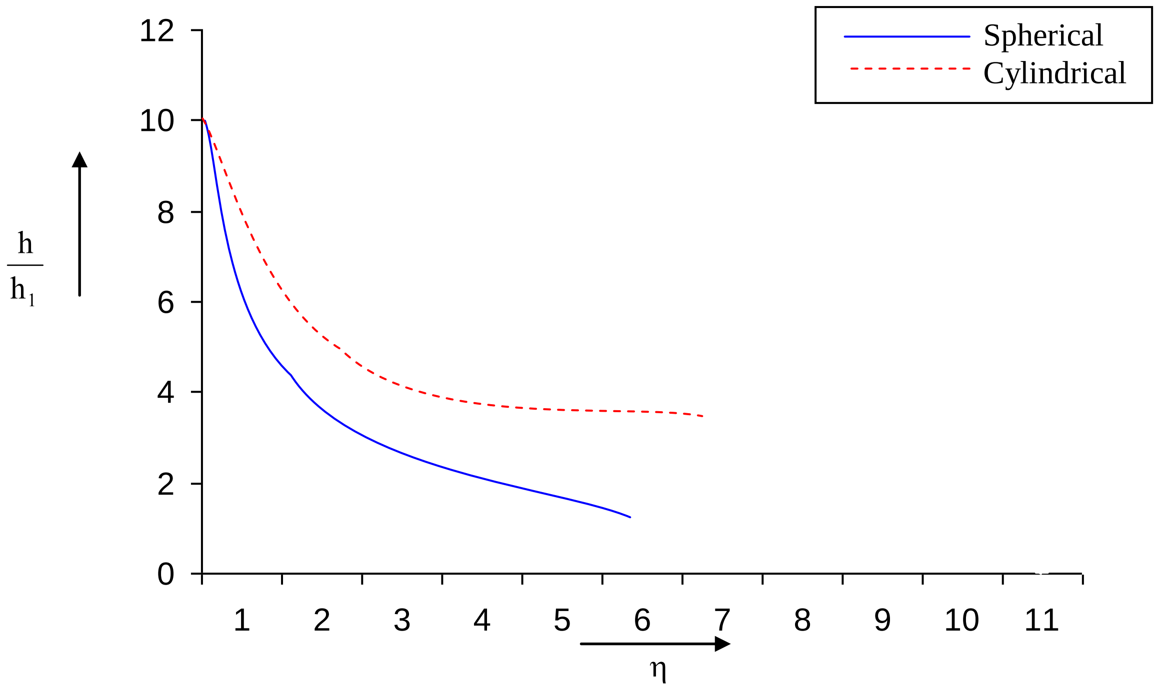
decrease as we move away from the shock surface where as velocity is minimum at the shock surface and increase as we move away from shock surface. We also see that variation in variable is more rapidly in spherical shock wave in comparison for cylindrical shock waves.



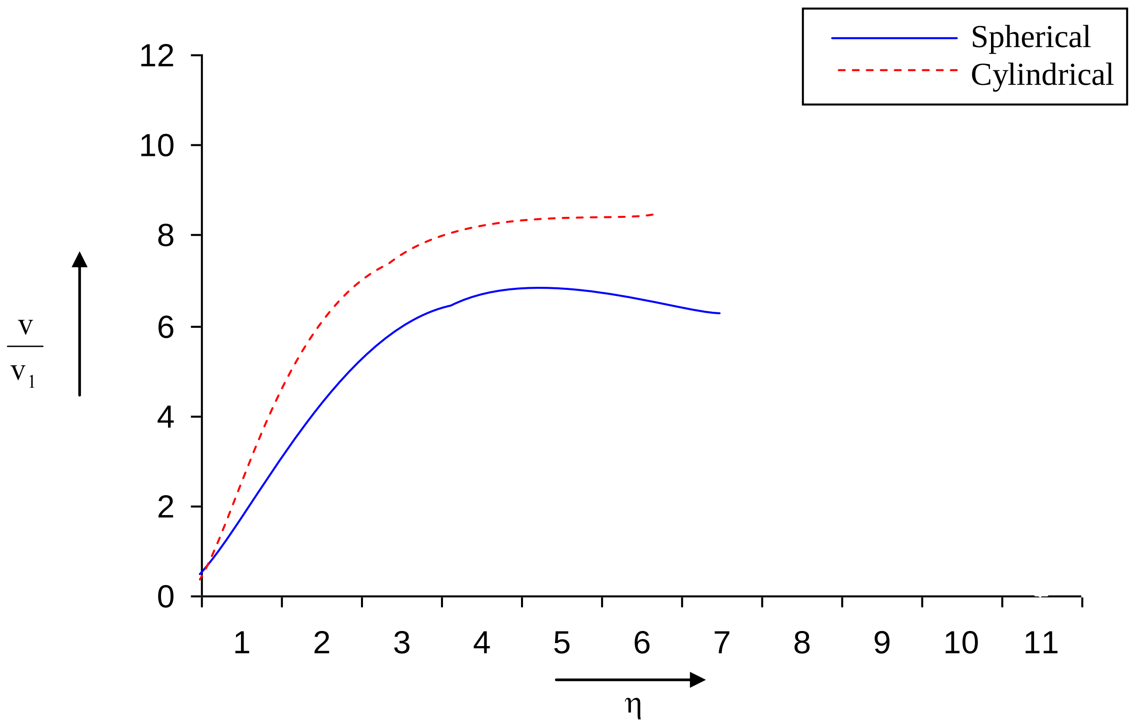
Pressure Distribution



Density Distribution



Magnetic Distribution



Velocity Distribution

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