Determination of the spherical aberration and the focal length of a concave mirror

I Vamvakas, V Bartzis and G Kompocholis

Technological Educational Institute of Athens, Department of Physics, Chemistry and Materials Technology, Agiou Spyridonos Street, 122 10 Aigaleo, Athens, Greece, e-mail I Vamvakas: ioannisvamvakas@gmail.com, e-mail V Bartzis: vbartzis@otenet.gr e-mail G Kompocholis: p.c.m.t@teiath.gr

Abstract
Geometrical Optics is an important scientific area of training of the undergraduate science students. It is usually restricted in lenses and mirrors of relatively small lens aperture in which the various errors are considered to be negligible. Our intent in this publication is the experimental approach and study of the   spherical aberration of the non-ideal concave mirror with a large aperture, in which the spherical aberration is not negligible. The focal length and the radius of curvature of the mirror are   also determined. The same experimental setup can be used to determine the focal length of ideal concave mirrors with negligible errors for an introductory level of geometrical optics.

Introduction
Mirror is every smooth and polished surface that reflects the light incident on it. Aspherical are called the mirrors whose reflective surface is a curved surface other than a sphere such as hyperbolic, elliptical and parabolic mirrors [1]. Spherical are called the mirrors whose reflective surface is a part of a sphere. Spherical mirrors are considerably easier and cheaper to fabricate than aspherical mirrors. Spherical mirrors are found in reflecting telescopes (Newtonian or Schmidt), in rear view mirrors and head lights of motor vehicles, in solar cookers and fishes' eyes [2]**.**
Aberrations are defects of mirrors and lenses in which rays of light parallel to but far from the optical axis (non-paraxial rays) are brought to a different focus from those close to the axis (paraxial rays). Spherical are called the aberrations when the incident light is monochromatic and they deteriorate the images making them unclear [3]. The spherical aberration of the eye and its correction has been attracting attention [4, 5].
In this article the spherical aberration of a concave mirror is investigated. The same experimental setup is used to determine the focal length and the radius of curvature of the mirror.

Ideal spherical mirror
The main characteristics of a spherical mirror (figure 1) are:

The centre O of the reflective surface is "the pole of the spherical mirror".
a) The centre C of the sphere the mirror is a part of is "the centre of curvature of the mirror".
b) The radius CO of the sphere is "the radius of curvature of the mirror".
c) The straight line through C and O is "the optical (or principal) axis of the mirror".

d) Every line passing through the centre of curvature of the mirror C and from a point D of the reflective surface of the mirror other than O is a "secondary axis of the mirror".

e) The angle ACB defined by the centre C of the spherical mirror and the end points A and B of the reflective surface is called the "aperture of the mirror". When the mirror has small aperture, it is considered ideal.
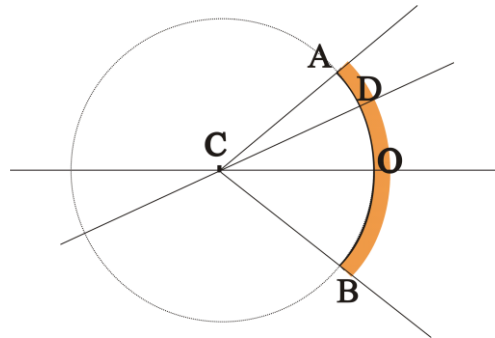
Figure 1. Spherical mirror. CO is the optical axis, CD is the secondary axis.

Ideal concave spherical mirror

Ideal spherical mirrors [6] with reflective surface the inner surface of the spherical shell are called "ideal concave spherical mirrors".

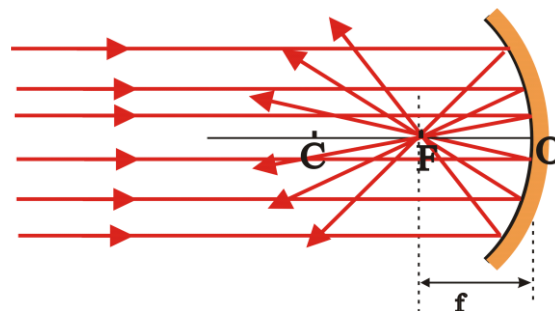Consider a light ray incident on the concave mirror parallel to the optical axis of the mirror (Figure 2).

Figure 2. The focus F of an ideal concave mirror.

After reflection the ray will pass through point F on the optical axis of the mirror, F is "the focal point" or "focus" of the mirror.

For an ideal mirror with small aperture we have approximately

$$FC = FO = R/2$$

where $R = CO$ is the radius of curvature of the mirror.

All light rays parallel to the optical axis of the mirror will pass through the focus F of the mirror after reflection (Figure 2). The distance between the focus F and the pole O of the mirror is called "the focal length of the mirror" $f = FO$ (Figure 2) and we have already mentioned that

$$f = \frac{R}{2} \tag{1}$$

Light rays incident on the mirror parallel to a secondary axis of the mirror converge to a single point F´ after reflection, "the secondary focus". If we consider the mirror aperture small, all these secondary foci are located on a plane called "the focal plane" (Figure 3).
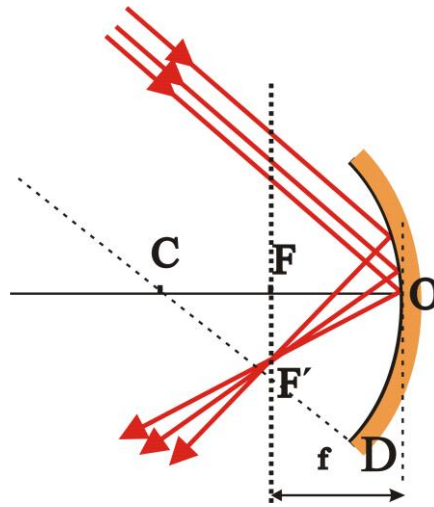

Figure 3. FF is the focal plane, CD is the secondary axis.

Spherical aberration of a concave mirror

The previous analysis of the spherical mirrors was based on the assumptions the mirror has small aperture therefore the incident rays are very close to its optical axis, this is the ideal mirror.

If the aperture is big, rays parallel to the optical axis of the mirror do not converge at the same point, "the focal point" or "focus", but they converge at the circumference of a circle (Figure 4).
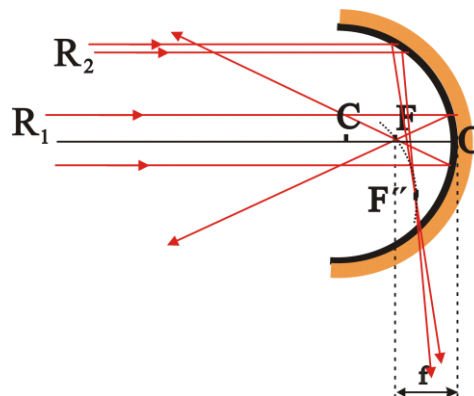

Figure 4. Convergence of paraxial rays R_1 and non-paraxial rays R_2 incident on a concave mirror.

In this case the image of a point source is not a single point but a spot, this is the spherical aberration of a mirror [7]. Beams of rays R_1 (paraxial) and R_2 (non-paraxial) in Figure 4 have different distances from the optical axis and they converge to different points F and F´´ respectively on the circumference of a circle.

Consider ray AB parallel to the optical axis at a distance d from it (Figure 5).
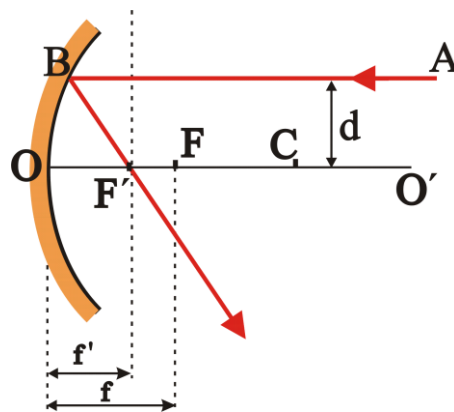

Figure 5. Non-paraxial ray AB intercepts the optical axis of the mirror at F' instead of the focus F.

After reflection ray AB will intercept the optical axis at the point F´ distant f´ from the pole O of the mirror. The relation between f', the radius of curvature R and the distance d is

$$\frac{f'}{R} = 1 - \frac{1}{2\sqrt{1-(\frac{d}{R})^2}} \qquad (2)$$

Equation 2 can be proved referring to Figure 6. BF' is the reflected ray of the non-paraxial ray AB, F' is the point the reflected ray BF' intercepts the optical axis. The second law of reflection [8] gives
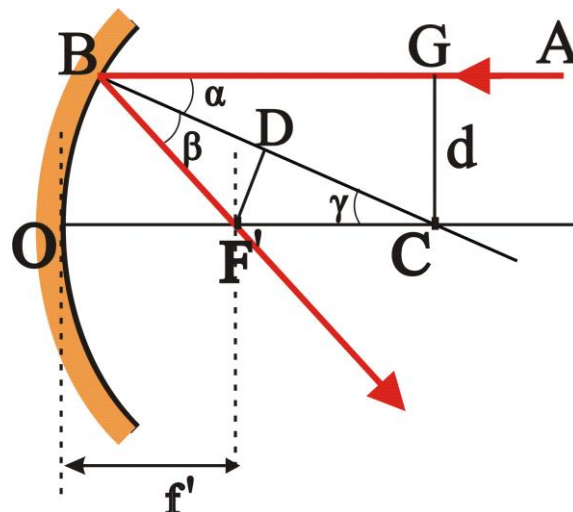
$$\alpha = \beta$$

Figure 6. Construction to prove 2 for the spherical aberration of a concave mirror.

and since AB and OC are parallel

$$\alpha = \gamma$$

Therefore the triangle $F'BC$ is isosceles and $F'D$ is both median and height, thus

$$DC = \frac{R}{2}$$

From the right-angle triangle $F'DC$ we obtain

$$\cos \gamma = \frac{R}{2F'C} \quad \Rightarrow \quad F'C = \frac{R}{2\cos\alpha} = \frac{R}{2\sqrt{1 - \sin^2\alpha}}$$

From the right-angle triangle CGB

$$\sin\alpha = \frac{d}{R}$$

The last two equations combined with

$$OF' = OC - F'C \quad \Rightarrow \quad f' = R - F'C$$

When $\dfrac{d}{R} \to 0$ then $\dfrac{f'}{R} = \dfrac{1}{2}$, therefore

$$f' = \frac{R}{2} \tag{3}$$

and the mirror is ideal.

Experimental setup
The experimental setup for measuring the focal length of a concave mirror consists (Figure 7 and 8) of a base B on which are placed a) the concave mirror M we want to determine the focal length, b) the principal axis OO′ of the mirror is engraved on the base B and it is graduated with a scale parallel to it, the beginning (the zero) of this scale coincides with the pole O of the mirror. c) The graduated scale KK' in the back of the mirror and parallel to it, the zero of this scale corresponds to the pole O of the mirror.
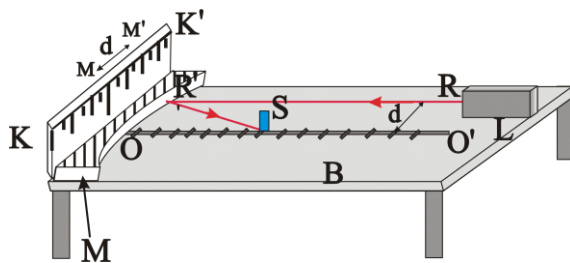

Figure 7. Sketch of the experimental setup.

Figure 8. Top view of the experimental setup.

d) The slider S (Figure 7) is able to move along the optical axis of the mirror. The path of the reflected ray can be drawn on the slider S; this is used for the alignment of

the laser. e) A laser source L with beam RR' parallel to the optical axis, is able to move in a direction perpendicular to the optical axis of the mirror.

When the system is aligned the position M corresponding to the pole O of the mirror is indicated on the scale KK´ (Figure 9).
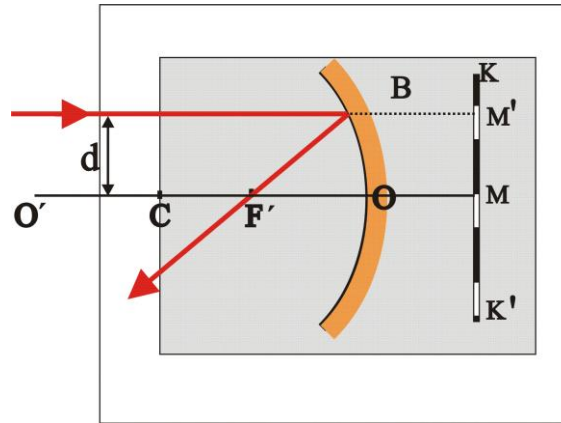

Figure 9. The scale KK' next to the concave mirror.

Position M corresponds to the distance d=0 of the beam from the optical axis of the mirror. Parallel translation of the laser source and its measurement are done with a sliding micrometer. The laser source is translated by distance d (Figure 9). M´ is the point the extrapolated ray of incidence intercepts the scale KK´. After reflection the ray will pass through point F´ on the optical axis of the mirror. This point can be easily marked with the help of the slider S and the distance OF´ can be measured with the graduated scale. Distance OF´ is the distance f' of (2).
Solving (2) for the radius of curvature of the mirror to a first order approximation one obtains

$$R = f' + \sqrt{f'^2 + \frac{1}{2}d^2} \qquad (4)$$

The focal length of the mirror f can be determined using (1).

Results and conclusions
The excellent agreement between the experimental data and the theoretical curve obtained from (2) is shown in figure 10.
The range of (d/R) is limited as seen by the experimental data in figure 10. This is due to the short range the micrometer is able to slide perpendicularly to the optical axis; work is under way to improve this.
The experimental setup provides a reliable and accurate enough method to determine the spherical aberration as well as the focal length and the radius of curvature of a non-ideal concave mirror with a large aperture. Also it indicates the identity between theoretical and experimental anticipation, proving the existing theory. It also measures the degree in which a concave mirror is closer to the ideal.
We observe when the fraction d/R increases the divergence between theoretical and experimental rates, it increases, also. The experimental setup, that we are using, it

limits the measurements of the bigger rates of the fraction d/R, something that we are going to solve with the construction of an appropriate experimental setup. Which it will enable to measure at the borders of the concave mirror, a paper that will be presented later.
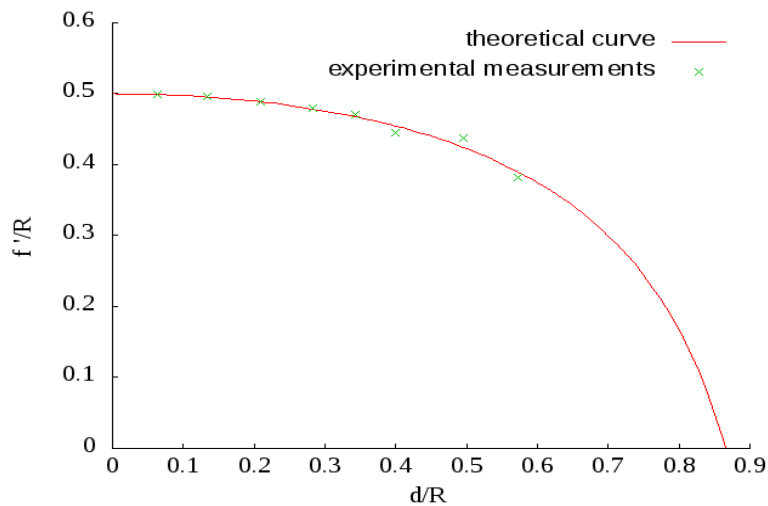

Figure 10. Plot of (f'/R) against (d/R) for a concave mirror. The solid line is the theoretical curve from (2), the discrete points are the experimental measurements.

The same experimental setup can be used to determine the focal length of ideal concave mirrors with negligible errors for an introductory level of geometrical optics. Determination of the same quantities of other curved mirrors and lenses with appropriate adjustments of the apparatus could also be investigated.

References

1. Hecht E and Zajac A 1974 Optics *Addison-Wesley Publishing Company Inc* p 123
2. Colicchia G et al 2009 The scallop's eye - a concave mirror in the context of biology Physics Education **44** pp 175–79
3. Hecht E and Zajac A 1974 Optics *Addison-Wesley Publishing Company Inc* p 175
4. Colicchia G and Wiesner H 2006 Measuring aberration of the eye with wavefront technology *Physics Education* **41** pp 307–10
5. Colicchia G 2007 Corneal reshaping and the prospect of 'super' vision *Physics Education* **42** pp 598-602
6. Hecht E and Zajac A 1974 Optics *Addison-Wesley Publishing Company Inc* p 124
7. Hecht E and Zajac A 1974 Optics *Addison-Wesley Publishing Company Inc* pp 175-176
8. Hecht E and Zajac A 1974 Optics *Addison-Wesley Publishing Company Inc* p 62