ON MACROSCOPIC TWO-PHASE ARTERIAL BLOOD FLOW THROUGH AN OVERLAPPING STENOSIS

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Abstract- The effects of an overlapping stenosis on blood flow characteristics in an artery have been investigated. To account for the presence of red cells, blood has been represented by a macroscopic two-phase model (i.e., a suspension of erythrocytes in plasma). The coupled differential equations describing the flow of fluid (plasma) and the particle (red cell) phases have been solved and the expressions for the flow characteristics, namely, the impedance, the wall shear stress, the shear stress at the stenosis throats and the shear stress at the critical height of the stenosis have been derived. It is shown that the impedance increases with the hematocrit as well as with the stenosis size. The shear stress at the two stenosis throats assumes the same magnitude. The shear stress at the stenosis critical height assumes significantly lower magnitude than its corresponding value at the throats. With respect to any given parameter, the nature of the variations of shear stresses at the throats and at the critical height of the stenosis is same as the flow resistance.
Key Words: Hematorit, impedance, shear stress, stenosis throats, stenosis critical height.

INTRODUCTION

The medical term “Stenosis or Arteriosclerosis” means narrowing of any body passage, tube or orifice (Young, 1979). Stenosis is an abnormal and unnatural growth that develops at various locations of the cardiovascular systems under diseased conditions, which occasionally results into serious consequences (Srivastava, 1995). The cause for the development of the frequently occurring cardiovascular disease, stenosis is related to the nature of blood movement and the mechanical behavior of the blood vessel walls. It is well known that the fluid dynamical parameters, particularly the high wall shear stress play an important role in the genesis of the disease, although the root causes of the formation of stenotic lesions are not well understood. Irrespective of the cause, it is well established that once the constriction has developed, it brings about the significant alterations in the blood flow, pressure distribution, wall shear stress and the flow resistance (impedance). In view of the possibility that haemodynamic factors play an important role in the genesis and proliferation of the disease has attracted the early investigators including Young (1968), Young and Tsai (1973), Deshpande et.al. (1976), Caro et.al. (1978), Ahmed and Giddens (1983), and several others to study the blood flow through local constrictions, after the first investigation of Mann et.al. (1938). A brief account of researches on the topic, reported so far, may be had from Young (1979), Srivastava (1995), Sarkar and Jayaraman (1998), Ku (1997), Mishra and Verma (2007), Mekheimer and El-Kot (2008), Srivastava and Rastogi (2009), etc.

Being a suspension of corpuscles, at low shear rates blood behaves like a non-Newtonian fluid (Merill et al., 1965; Charm and Kurland, 1965, 1974; Hershey, et al., 1964; Huckaba and Hahu, 1968). However, the theoretical study of Haynes (1960) and experimental observations of Cokelet (1972) indicate that blood can not be treated as a single-phase homogeneous viscous fluid in narrow arteries (of diameter $\leq 1000\mu$m). Skalak (1972) concluded that an accurate description of flow requires consideration of erythrocytes (red cells) as discrete particles in small arteries. Srivastava and Srivastava (1983) observed that the individuality of red cells (of diameter 8μm) is important in such large vessels with diameter up to 100 cells diameter. A brief survey of the literature on multiphase blood flow has recently been presented by Srivastava (2007). On the other hand, a survey of the literature on arteriosclerotic development reveals that the studies conducted are mainly concerned with the single symmetric and non-symmetric stenoses. However, the stenosis may develop in series (multiple stenoses) or may be of irregular shapes. Assuming the pressure variation only along the axis of the tube, Chakravarty and Mandal (1994) studied the effects of an overlapping stenosis on arterial flow problem of blood. An attempt is made in the present investigation to explore the effects of an overlapping stenosis on the flow characteristics of blood taking into account that blood flowing is to be treated as macroscopic two-phase fluid (i.e., a suspension of erythrocytes in plasma). The wall in the vicinity of the stenosis is usually relatively solid when stenoses develop in the living vasculature. The artery length is considered large enough as compared to its radius so that the entrance, end and special wall effects can be neglected.

FORMULATION OF THE PROBLM

Consider the axisymmetric flow of blood through an artery of circular cross section with an overlapping stenosis specified at the position shown in Fig.1. The geometry of the stenosis which is assumed to be manifested in the arterial wall segment is described (Chakravarti and Mandal ,1994) as

$$\frac{R(z)}{R_0}=\begin{cases}1-\dfrac{3}{2}\dfrac{\delta}{R_0L_0^4}\left[11(z-d)L_0^3-47(z-d)^2L_0^2+72(z-d)^3L_0-36(z-d)^4\right] \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad d\le z\le d+L_0 \\ 1, \quad otherwise\end{cases} \tag{1}$$

where $\{R(z),R_0\}$ are the radius of the tube (with, without) stenosis, L_0 is the length of the stenosis and d indicates its location, δ is the maximum height of the stenosis into the lumen, appears at the locations: $z=d+L_0/6$ and $z=d+5L_0/6$. The height of the stenosis at $z=d+L_0/2$ is called critical height, is $3\delta/4$.

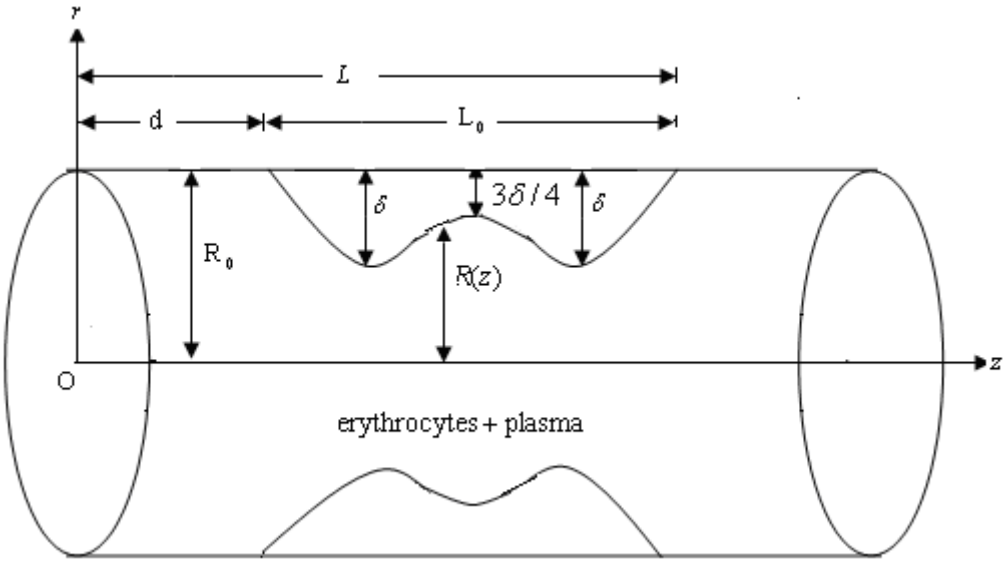

Fig.1The flow geometry of an arterial overlapping stenosis.

Blood is assumed to be represented by a two-phase macroscopic model, that is, a mixture of plasma and erythrocytes (red cells). Due to the complicated structure of blood and the circulatory system, to analyse the problem in an exact manner seems to be a formidable task. Thus, under the simplified assumptions along with their justifications (Srivastava and Srivastava, 1983), the equations describing the steady flow of a two-phase macroscopic model of blood may be expressed as

$$(1-C)\,\rho_f\left\{u_f\frac{\partial u_f}{\partial z}+v_f\frac{\partial u_f}{\partial r}\right\}=-(1-C)\frac{\partial p}{\partial z}+(1-C)\,\mu_s(C)\,\nabla^2 u_f + CS(u_p-u_f), \tag{3}$$

$$(1-C)\,\rho_f\left\{u_f\frac{\partial v_f}{\partial z}+v_f\frac{\partial v_f}{\partial r}\right\}=-(1-C)\frac{\partial p}{\partial r}+(1-C)\,\mu_s(C) \times\left(\nabla^2-\frac{1}{r^2}\right)v_f + CS(v_p-v_f), \tag{4}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\,(1-C)\,v_f\right]+\frac{\partial}{\partial z}\left[(1-C)u_f\right]=0, \tag{5}$$

$$C\,\rho_p\left\{u_p\frac{\partial u_p}{\partial z}+v_p\frac{\partial u_p}{\partial r}\right\}=-C\frac{\partial p}{\partial z}+CS(u_f-u_p), \tag{6}$$

$$C\,\rho_p\left\{u_p\frac{\partial v_p}{\partial z}+v_p\frac{\partial v_p}{\partial r}\right\}=-C\frac{\partial p}{\partial r}+CS(v_f-v_p), \tag{7}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\,C\,v_p\right]+\frac{\partial}{\partial z}\left[C u_p\right]=0, \tag{8}$$

with $\nabla^2 \equiv (1/r)\,\partial/\partial r\,(r\,\partial/\partial r)+\partial^2/\partial z^2$ as Laplacian operator, r and z are the cylindrical polar coordinate system with z measured along the tube axis and r perpendicular to the axis of the tube. (u_f, u_p) and (v_f, v_p) are the axial and radial

components of the (fluid, particle) velocities, ρ_f and ρ_p are the actual density of the material constituting the fluid (plasma) and the particle (erythrocyte) phases, respectively, (1-C) ρ_f is the fluid phase and $C\rho_p$ is particle phase densities, C denotes the volume fraction density of the particles, p is the pressure, $\mu_s(C) \simeq \mu_s$ is the mixture viscosity (apparent or effective viscosity) , S is the drag coefficient of interaction for the force exerted by one phase on the other, and the subscripts f and p denote the quantities associated with the plasma (fluid) and erythrocyte (particle) phases, respectively. The volume fraction density of the particles, C is chosen to be constant which is a good approximation for the low concentration of small particles (Drew, 1979; Srivastava et al., 1994). Others limitations of the present model are the same as discussed in Srivastava and Srivastava (2009). Now following the reports of Young (1968), Srivastava and Rastogi (2009), the equations governing the laminar, steady, one-dimensional flow of blood in an artery, under the conditions of a mild stenosis (Young, 1968: Srivastava and Rastogi, 2009a): $\delta / R_0 << 1$, $R_e(2\delta / L_0 << 1)$ and $2R_0 / L_0 \sim O(1)$, are derived (Drew, 1979; Srivastava and Srivastava, 2009) from equations (3)-(8) as

$$(1-C)\frac{dp}{dz} = (1-C)\frac{\mu_s(C)}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})u_f + Cs(u_p - u_f), \tag{9}$$

$$C\frac{dp}{dz} = Cs(u_f - u_p). \tag{10}$$

The expressions for drag coefficient of interaction, S and the viscosity of the suspension, μ_s for the present study are selected (Srivastava and Srivastava, 2009) as

$$S = 4.5\left(\frac{\mu_0}{a_0^2}\right)\frac{\{4+3(8C-3C^2)^{1/2}+3C\}}{(2-3C)^2} \tag{11}$$

$$\mu_s(C) \approx \mu_s = \frac{\mu_0}{1-mC},$$

$$m = 0.07\exp[2.49C + (1107/T)\exp(-1.69C)] \tag{12}$$

where T is the measure in absolute scale of temperature (K) , μ_0 is the constant plasma viscosity and a_0 is the radius of a particle (red cell).

The boundary conditions are

$$u_f = 0 \quad at \quad r = R(z) \tag{13}$$

$$\frac{\partial u_p}{\partial r} = 0 \quad at \quad r = 0 \tag{14}$$

ANALYSIS

An integration of equations (9) and (10), subject to the boundary conditions (13) and (14), yields the following expressions for the velocity of fluid and particle phases as

$$u_f = -\frac{R_0^4}{4(1-C)\mu_s}\frac{dp}{dz}\left\{(R/R_0)^2-(r/R_0)^2\right\}, \tag{15}$$

$$u_p = -\frac{R_0^4}{4(1-C)\mu_s}\frac{dp}{dz}\left\{(R/R_0)^2-(r/R_0)^2+\frac{4(1-C)\mu_s}{SR_0^2}\right\}, \tag{16}$$

The flow flux, Q is now calculated as

$$Q = 2\pi(1-C)\int_0^R ru_f dr + 2\pi C\int_0^R ru_p dr$$

$$= -\frac{\pi R_0^4}{8(1-C)\mu_s}\frac{dp}{dz}\left\{(R/R_0)^4+\beta(R/R_0)^2\right\}, \tag{17}$$

where $\beta = 8C(1-C)\mu_s / SR_0^2$, a non –dimensional suspension parameter.

One obtains from equation (17) now

$$\frac{dp}{dz} = -\frac{8(1-C)\mu_s}{\pi R_0^4}\phi(z), \tag{18}$$

with

$$\phi(z) = \frac{1}{(R/R_0)^4+\beta(R/R_0)^2}.$$

The pressure drop, $\Delta p(= p \ \ at \ \ z=0 \ \ and \ \ -p \ \ at \ \ z=L)$ across the stenosis in a tube of length L is calculated from equation (18) as

$$\Delta \mathrm{p} = \int_0^{\mathrm{L}}\left(-\frac{\mathrm{dp}}{\mathrm{dz}}\right)\mathrm{dz} = \frac{8(1-C)\mu_s Q}{\pi R_0^4}\psi, \tag{19}$$

where

$$\psi = \int_0^L \phi(z)dz = \int_0^d [\phi(z)]_{R/R_0=1}dz + \int_d^{d+L_0}[\phi(z)]dz + \int_{d+L_0}^L [\phi(z)]_{R/R_0=1}dz. \tag{20}$$

The first and the third integrals in the expression for ψ obtained above are straight forward whereas the analytical evaluation of the second integral is almost a formidable task and thus will be evaluated numerically. Following now Srivastava (1995) and Srivastava and Rastogi (2009a), one obtains the expressions for impedance λ, the wall shear stress in the stenotic region, τ_w, the shear stress at stenosis throats, τ_s and the shear stress at the critical height of the stenosis, τ_c, in their non-dimensional form as

$$\lambda = (1-C)\mu\left\{\frac{1-L_0/L}{1+\beta}+\frac{1}{L}\int_d^{d+L_0}\frac{dz}{(R/R_0)^4+\beta(R/R_0)^2}\right\}, \tag{21}$$

$$\tau_w = \frac{(1-C)\mu}{(R/R_0)^3+\beta(R/R_0)}, \tag{22}$$

$$\tau_s = \frac{(1-C)\mu}{(1-1.25\,\delta/R_0)^3+\beta(1-1.25\,\delta/R_0)}, \tag{23}$$

$$\tau_c = \frac{(1-C)\mu}{\{1-0.75\delta/R_0\}^3+\beta(1-0.25\delta/R_0)}, , \tag{24}$$

where $\lambda = \dfrac{\bar{\lambda}}{\lambda_0},\ (\tau_w, \tau_s, \tau_c) = \dfrac{(\bar{\tau}_w, \bar{\tau}_s, \bar{\tau}_c)}{\tau_0},$

$$\bar{\lambda} = \frac{\Delta p}{Q},\ \bar{\tau}_W = -\frac{R}{2}\left(\frac{dp}{dz}\right),$$

$$\bar{\tau}_S = \left[-\frac{R}{2}\left(\frac{dp}{dz}\right)\right]_{R/R_0} = \left(1-1.25\frac{\delta}{R_0}\right),$$

$$\bar{\tau}_c = \left[-\left(\frac{R}{2}\right)\frac{dp}{dz}\right]_{R/R_0} = \left(1-0.75\frac{\delta}{R_0}\right),\ \mu = \frac{\mu_s}{\mu_0},$$

$$\lambda_0 = \frac{8\mu_0 L}{\pi R_0^4},\ \tau_0 = \frac{4\mu_0 Q}{\pi R_0^3},$$

λ_0 and τ_0 are the flow resistance and wall shear stress for a normal artery (no stenosis) in the absence of the particle phase (i.e. C=0, Newtonian fluid).

In the absence of the particles (i.e. C=0) the results for a Newtonian fluid are derived from Equation (21)-(24), as

$$\lambda_N = 1 - \frac{L_0}{L} + \frac{1}{L}\int_d^{d+L_0} \frac{dz}{(R/R_0)^4}, \tag{25}$$

$$\tau_{wN} = \frac{1}{(R/R_0)^3}, \tag{26}$$

$$\tau_{sN} = \frac{1}{\left(1-1.25\frac{\delta}{R_0}\right)^3}, \tag{27}$$

$$\tau_{cN} = \frac{1}{\left(1-0.75\frac{\delta}{R_0}\right)^3}, \tag{28}$$

NUMERICAL RESULTS AND DISCUSSIONS

To discuss the results of the study quantitatively, computer codes are developed to evaluate analytical results obtained in equations (21)-(24) at the temperature of 37^0C in a tube of radius 0.01cm for various parameter values selected from Young (1968), Srivastava (1995) and Srivastava and Rastogi (2009a). The parameter values are: d(cm) = 0; L_0 (cm) = 1; L(cm) = 1, 2, 5 ; C = 0, 0.2, 0.4, 0.6; δ / R_0=0, 0.05, 0.10, 0.15, 0.20. It is to note that present study corresponds to the case of a Newtonian fluid and no stenosis for parameter values C=0 and δ / R_0=0, respectively.

The impedance (resistance to flow), λ increases with hematocrit, C for any given stenosis height, δ / R_0 and the flow characteristic, λ increases with stenosis height, δ / R_0 for any given hematocrit, C (Fig.2). The blood flow characteristic, λ decreases with increasing tube length for any set of other parameters given (Fig.3), which in turns implies that λ increases with the stenosis length

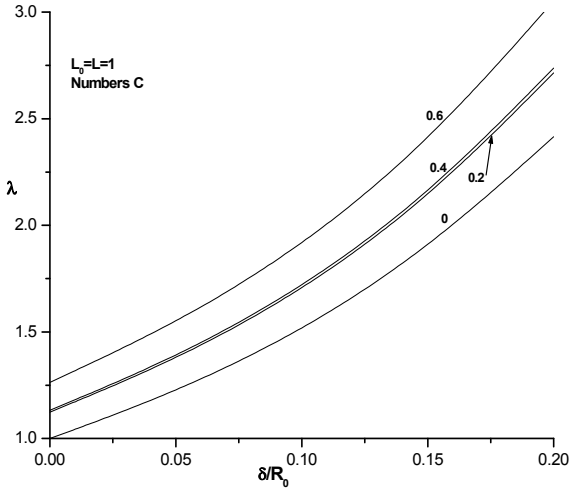

Fig.2 Variation of impedance,λ with δ/R_0 for different C.

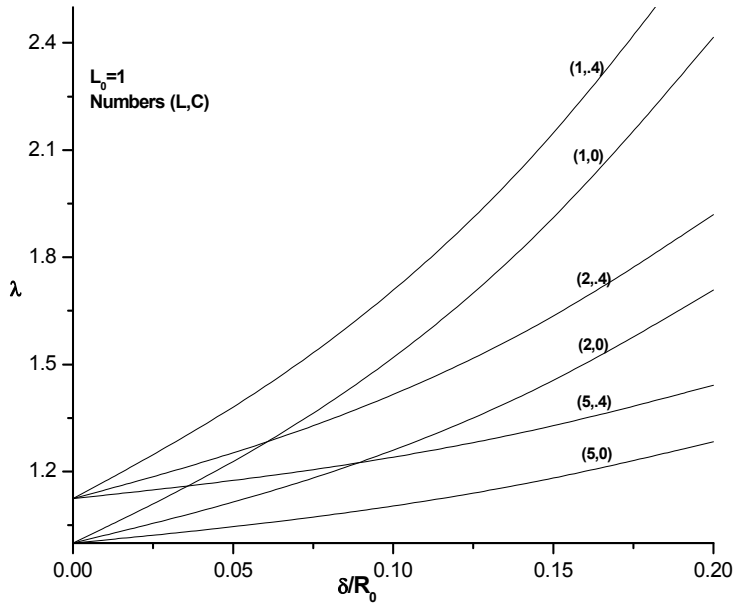

Fig.3 Variation of impedance,λ with δ/R_0 for different L and C.

L_0. The flow resistance, λ is seen steeply increasing with increasing hematocrit, C for any given

stenosis height, δ / R_0 (Fig.4).

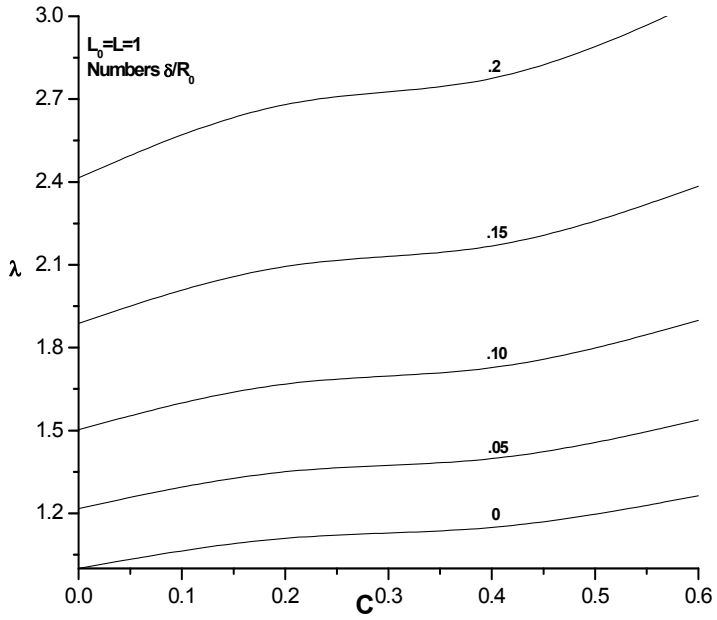

Fig.4 Variation of impedance,λ with C for different δ/R_0.

The wall shear stress distribution in the stenotic region, τ_w assumes higher magnitude for higher stenosis height, δ / R_0 and also for higher hematocrit, C. The

flow characteristic, τ_w rapidly increases from its approached value at $z = 0$ to its peak value in the upstream of the first stenosis throat at z/L_0=0.167, it then decreases steeply in the downstream of the first throat to its magnitude at the critical height ($0.75\,\delta / R_0$) of the stenosis at $z = L_0/2$. τ_w further increases steeply in the upstream of the second stenosis throat and attains its peak magnitude (same as at the first throat) at the second stenosis throat at $z = 0.833$, and then decreases rapidly to the same magnitude as its approached value at the end point of the constriction profile at z/L_0=1. As expected, the shear stress at two stenosis throat has the same magnitude. One notices that for any given set of parameters, there occurs a significant difference in the magnitude of the shear stresses at the stenosis throats and at the critical height of the stenosis (Fig.5). One observes that the flow characteristic, shear stress at stenosis throats τ_s increases with hematocrit, C and the stenosis size (height & length).

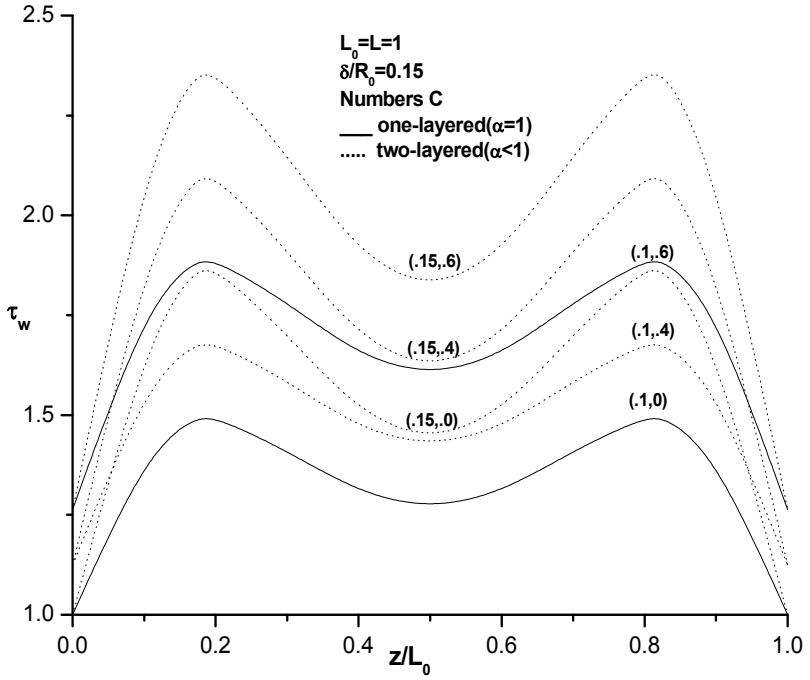

Fig.5 Wall shear stress distribution in stenotic region, τ_w for different C.

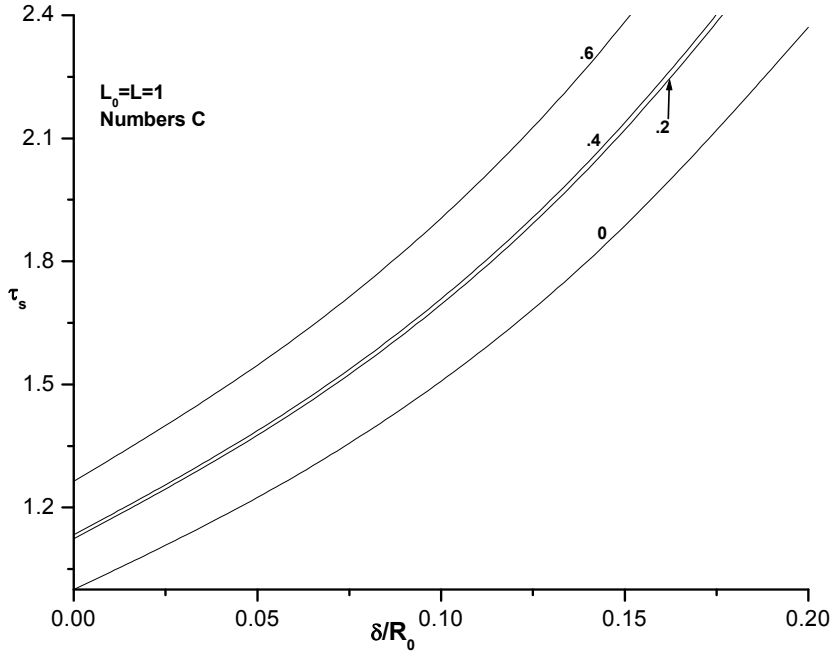

Fig.6 Variation of shear stress at stenosis throats, τ_s with δ/R_0 for different C.

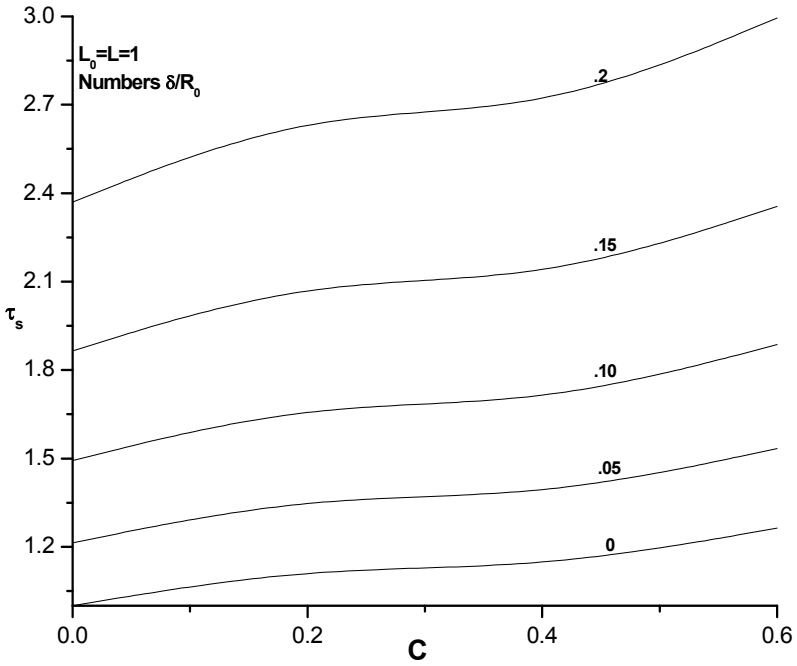

Fig.7 Variation of shear stress at stenosis throats, τ_s with C for different δ/R_0.

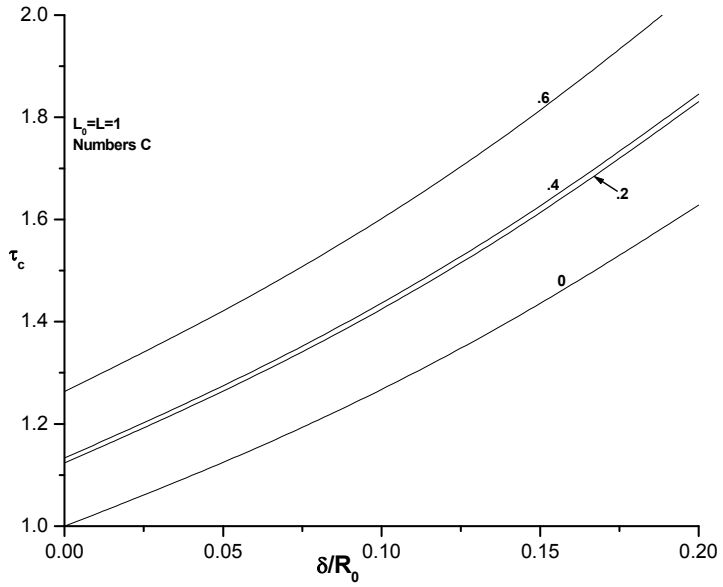

Fig.8 Variation of shear stress at critical stenosis height, τ_c with δ/R_0 for different C.

With respect to any given parameter, the nature of the variations of τ_s is similar to that the impedance, λ (Fig 2 and 6). The magnitude of the shear stress, τ_s is higher than the corresponding magnitude of the impedance, λ (Fig.7). The nature of the variations of the shear stress at the critical stenosis height (at $z/L_0=0.5$), τ_c is similar to that of the flow resistance, λ and shear stress at stenosis throats, τ_s. One notices that the flow characteristic, τ_c assumes significantly lower magnitude than the corresponding values of both λ and τ_s (Figs. 8 and 9).

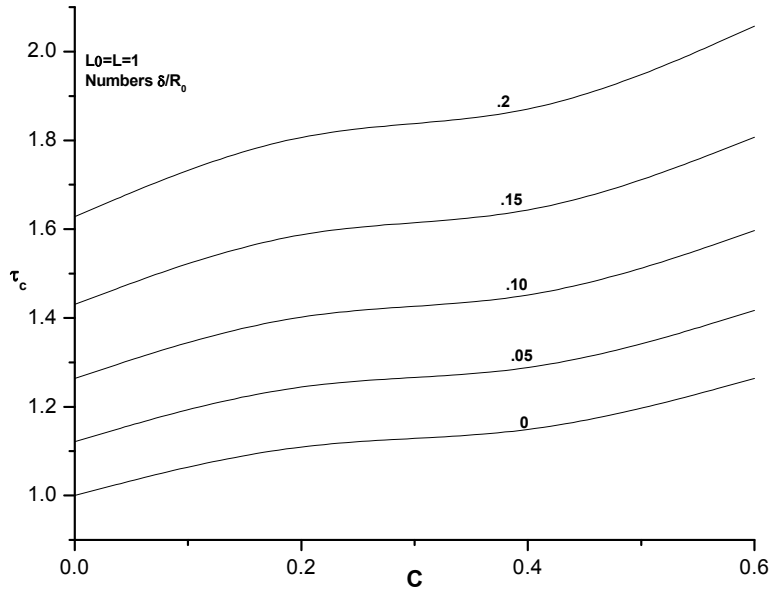

Fig.9 Variation of shear stress at stenosis critical height, τ_c for with C for different δ/R_0.

The discussions presented above clearly reveals the significance of the present study. The two-phase fluid seems to be more sensitive to the stenosis than the single phase fluid (i.e., C=0, Newtonian fluid). The condition $\delta / R_0 << 1$, limits the usefulness of the present study to very early stages of vessel constrictions, which enables one to use the fully developed flow equations and leads to the locally Poiseuille like flow and closed form solutions. Use of the parameter, δ / R_0 is restricted to the values up to 0.15 as beyond this value a separation in the flow may occurs even at a relatively small Reynolds number (Young, 1968; Srivastava, 1995). Although, the study has been conducted under various restrictions and simplification, it still enables one to observe the effects of red cells on the flow characteristics due to an overlapping stenosis in arteries.

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