A Simple Model for Ski Jump Flight Mechanics Used as a Tool for Teaching Aircraft Gliding Flight

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Abstract

A teaching tool is presented in the form of a spreadsheet mathematical model of a ski jumper in free flight. The aim of the simulation exercise is to introduce students taking courses in basic flight mechanics, to fundamental theoretical concepts and to simple simulation techniques. Using an example from the field of sports can stimulate student interest and enhance their motivation. It is concluded that the model can easily be implemented and correctly simulates the physical principles of ski jumping free flight and can therefore fulfill its purpose of enhancing the teaching of a key topic in the field of flight mechanics.

***Keywords-** flight mechanics; model; ski jump; gliding flight*

I. Introduction

The author of this paper teaches the core subject of Flight Mechanics, at the Department of Aircraft Technology, at the Technological Educational Institute (TEI) of Chalkis in Greece. A key topic covered in most flight mechanics courses is that of gliding flight which is concerned with the aircraft motion without the assistance of a propulsion system. The main parameters of interest for this flight regime are aircraft range (the horizontal distance traveled), rate of descent, and gliding ratio (the ratio of the range over the vertical drop in height). The theory leading to the derivation of the above parameters is covered in detail in most relevant textbooks (for example, Yechout et al [1]).

Furthermore, the development of mathematical models leading to trajectory simulation forms an essential component of many real life applications in the aerospace field. It is thus important that students are introduced at an early stage of their studies, to the use of basic modeling techniques, particularly to those with a relevance to flight simulation.

The use of sport as a means of increasing student understanding and motivation has been proposed by several authors (for example, Ireson [2]). Using examples from sporting events to illustrate basic physical principles can constitute an effective means of teaching. It is the purpose of this paper to present one such case, where a relatively simple model can easily be implemented by students in a spreadsheet environment in order to simulate the free flight phase of a ski jump, demonstrating the principles of aircraft gliding flight.

A concise overview of ski jumping and the physics behind it is provided by Banks [3] and de Mestre [4]. The sport constitutes one of the most popular Winter Olympics events with its rules set by the International Ski Federation (http://www.fis-ski.com/).

A ski jump essentially consists of the following five stages: the in-run (where the skier gathers momentum by descending a slope, take off (where the skier springs forward in order to attain the free flight position), free flight (where the skier “flies” through the air), the landing, and the out-run (where the skier decelerates until his final stop).

For the purpose of emulating aircraft gliding flight, it is the free flight stage that is of greatest interest. During this stage of the jump the athlete essentially glides through the air with the aim of maximizing the distance traveled. Throughout this flight, as pointed out by Banks [3], the skier tries to maintain a constant angle of attack, i.e. a constant incidence with respect to his velocity vector. The assumption of constant angle of attack simplifies the modeling exercise, as illustrated in the following section.

II. the model

It is assumed, that when in free flight, a skier is under the influence of three forces: the weight, the aerodynamic drag and lift forces, as shown in Fig. 1. Also of importance, are the angle of attack of the skier (incidence relative to the velocity vector) and the flight path angle (velocity vector inclination to the horizontal, taken here as positive when the velocity vector is inclined beneath the horizontal).

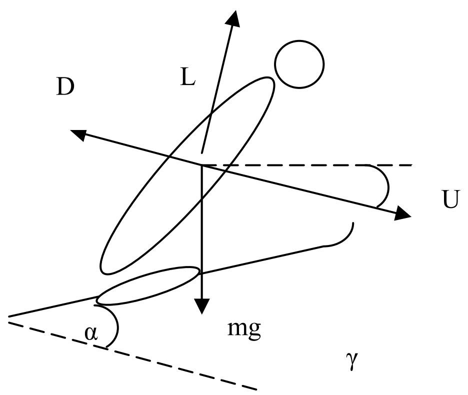

Figure 1. Forces acting on a skier in free flight

Assuming that the weight of the skier (and his apparatus) is constant and equal to the gravitational force, it is the two aerodynamic forces that require further attention.

Mathematically the drag and lift acting on the skier are derived from the following formulas:

$$D = \frac{1}{2}\rho A C_D U^2 \quad (1)$$

$$L = \frac{1}{2}\rho A C_L U^2 \quad (2)$$

Where,
D is the drag force, L is the lift force, ρ is the air density, A is the effective area C_D is the drag coefficient, C_L is the lift coefficient. U is the total speed

Taking the effective area as being 0.7 m2 , the mass of the skier as 75 kg and the density of the air to be 1.17 kg/m3, it is necessary to choose an aerodynamic model that will define the values of drag and lift coefficients. The chosen model for this application is that of (Krylov & Remizov[5]), who by use of wind tunnel tests arrived at the following expressions:

$$C_L = -0.00025\alpha^2 + 0.0228\alpha - 0.092 \quad (3)$$

$$C_D = 0.0103\alpha \quad (4)$$

Where, α is the angle of attack

It is evident from equations (3), (4), that the aerodynamic coefficients, for a fixed skier-ski configuration, are assumed to be a function of angle of attack only. As the aim of the model is to provide insight for pedagogical purposes, it is simplicity rather than accuracy which is the top priority. It is also worth noting that the three forces used to model the dynamics of a ski jumper in free flight are the same ones used to model an aircraft in gliding flight.

Having obtained expressions for all forces driving the flight of the athlete, it is then possible to derive the equations of motion, which in scalar form are:

$$m\frac{dU}{dt} = -\frac{1}{2}\rho AC_D U^2 + mg\sin\gamma \quad (5)$$

$$m\frac{d\gamma}{dt}U = -\frac{1}{2}\rho AC_L U^2 + mg\cos\gamma \quad (6)$$

Where, m is the skier mass, g is the gravitational constant, γ is the flight path angle

The above differential equations have no closed form solutions and are solved by implementing a simple Euler routine, ideally suited to spreadsheet software. Furthermore, this type of routine is easily understood by students with no prior knowledge of numerical techniques for solving differential equations.

III. simulation results

With suitable supervision, students are able to set up the simple model described previously, in a spreadsheet environment. As inputs, they are able to define the initial speed, flight path angle and angle of attack (assumed constant throughout the flight). A slider can be used to alter the angle of attack in order to directly view the effect on the resulting trajectory.

Results produced from the simulation are presented in Fig. 2-4. Trajectory coordinates are referred to the takeoff point and in this case are those obtained for a jump of 100 m horizontal distance, with an initial flight path angle of 10 deg, an angle of attack of 30 deg and initial speed of 28 m/s. It is noted that for this length of jump the vertical drop is approximately 70 m which is of the order observed for actual jumps of this type. This gives a glide ratio of approximately 1.43 and a total jump length of approximately 122 m.

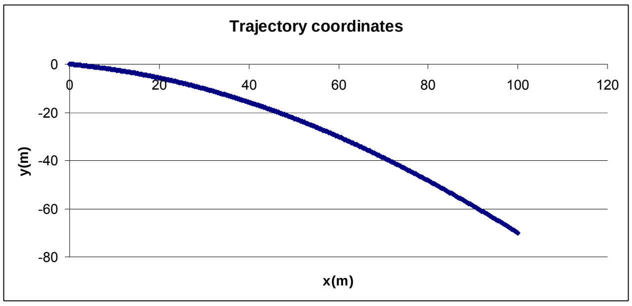

Figure 2. Trajectory of a skier in free flight

In Fig. 3 the speed variation during the flight is presented. There is an increase due to the effect of gravity which results in a landing velocity of approximately 41 m/s. Once again, although slightly high, this is consistent with previous studies on ski jumps (for example, Ward-Smith & Clements [6]).

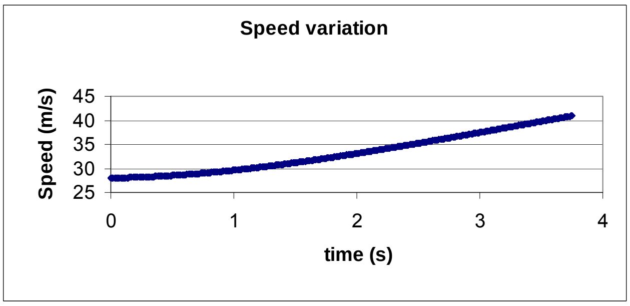

Figure 3. Speed of a skier in free flight

In order to asses the influence of the aerodynamic forces involved, those are presented in Fig. 4-5. It is noted that both forces increase with time due to the increase of the total speed. More importantly, both aerodynamics forces are seen to always be greater than 13% of the weight of the skier and thus are rightly included in our simulation module.

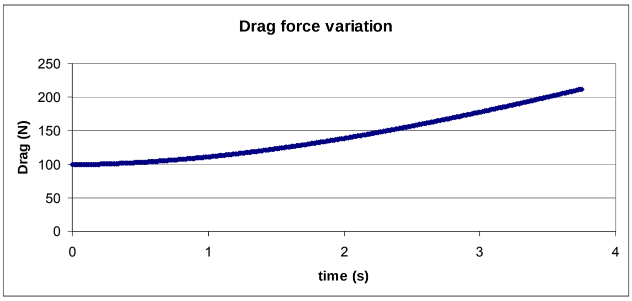

Figure 4. Drag force acting on a skier in free flight

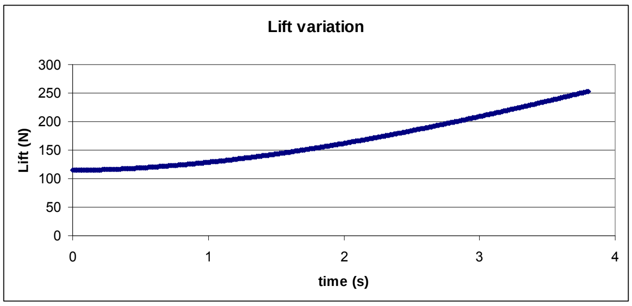

Figure 5. Lift force acting on a skier in free flight

For students studying the subject of flight mechanics, a familiar quantity is the, so called, load factor (n), the ratio of the lift force to the weight of the aircraft. In Fig. 6, the variation of load factor is presented, confirming that the lift acting on the skier can reach values of approximately 35% of the weight.

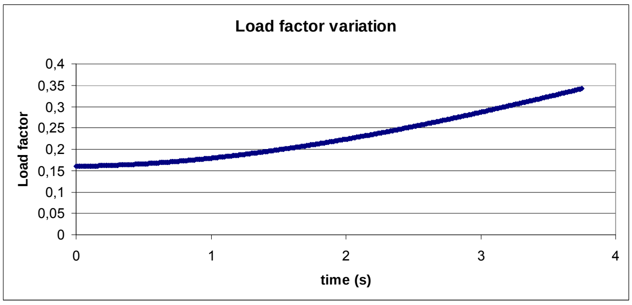

Figure 6. Load factor of a skier in free flight

Finally, in Fig. 7 the variation of the flight path angle is presented and it is observed that the landing angle is approximately 49 deg. Note that the skier will land on a slope that has a 36 deg inclination, so the angle of approach relative to the ground will be approximately 13 deg. Once again, this is consistent with the results from previous studies.

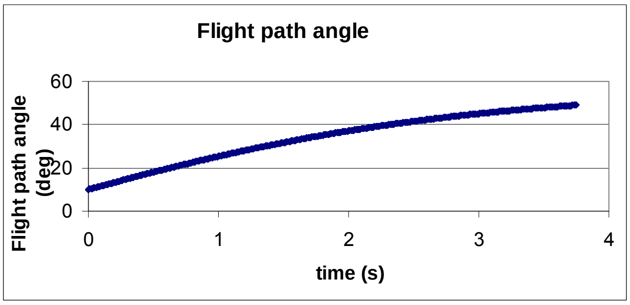

Figure 7. Flight path angle of a skier in free flight

There are several features of the model that can be of pedagogical value to students. The following is one such example: By varying the angle of attack, the students can study the effect on the glide ratio, an important parameter when dealing with aircraft flight mechanics, as described in most textbooks. In fact, it can be shown that for an aircraft in a steady descent, i.e., with a velocity constant in magnitude and direction, the glide ratio is equal to the Lift/Drag ratio of the aircraft. Although there will exist a relationship between glide ratio and Lift/Drag ratio for the 100m (in horizontal distance) jump of a skier, those will not be equal, as for the duration of the flight the skier will be accelerating and not in a steady descent. This is an important point that should illustrate to students that care should be taken when applying textbook theory to real life applications. In Table 1 glide ratios and Lift/Drag ratios are provided for various values of skier angle of attack, for jumps resulting in a horizontal distance travelled of 100 m and with the same initial conditions (other than the angle of attack), as those provided previously. It is noted, that although a maximum Lift/Drag ratio is obtained at an angle of attack of approximately 20 deg, the maximum glide ratio is achieved at approximately 35 deg.

TABLE I. Glide ratio and Lift/Drag ratio variation with α

α (deg)	Glide ratio	Lift/Drag
10	1.28	1.08
20	1.38	1.28
30	1.43	1.19
40	1.43	1.02
50	1.38	0.82

Other aspects of flight mechanics that can be tackled by use of the model, are the variation of Lift/Drag ratio with angle of attack (and the existence of an optimum angle), the concept of trimmed flight (flight for which the net linear and angular accelerations are zero), gliding flight endurance (total time of flight), rate of descent, etc. Once again, the parallels drawn between skier free flight and aircraft gliding flight can highlight the point that the same physical principles and modelling techniques can be applied to a great variety of situations.

IV. conclusions

A simple model has been presented that simulates the dynamics of a skier in free flight. The model can easily be implemented by students providing them with an introduction to numerical modelling methods.

Simulation results are consistent to the physical principles governing ski jumping and to previous studies. The model features can be used to teach important principles in the field of flight mechanics.

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