TEACHING THE NATURAL NUMBERS AS OPERATORS

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Abstract

Children can construct the concept of the natural number sufficiently, if they are engaged in activities concerning additive as well as multiplicative structures. Kindergarten children's engagement with activities concerning the number as operator has a special research interest, as there is not a lot of work on it. In this workshop the activities discussed were part of a broader research program, which aims to the study of changes that occur to kindergarten's children conception about natural number, when activities referring to the number as operator are proposed.

Keywords: kindergarten, mathematics, operators, multiplication

Introduction

This workshop is a presentation at the CIEAM55 (2003). That is an International Conference of Didactic of Mathematics which has implemented at the Poland. The aim of the workshop was that to collaborate with the participants on ideas concerning the approach of the concept of natural number as operator with appropriate activities in early childhood.

Theoretical Index

The natural number as operator is affiliated with the concept of multiplication and division and it is mathematically connected with the multiplicative structures. More specifically:

a. The natural number as operator works as a "machine" transforming a quantity to another one (multiplying factor) (Yamanoshita & Matsushita,1996).
b. The natural number as operator activating on natural numbers has as result always natural numbers.
c. The natural number as operator is a dimensionless quantity (Nesher,1988-Vergnaud,1983)
d. The natural number as operator does not change the nature of the referent of the quantity on which it operates (Schwartz,1988)
e. The natural number as operator changes the magnitude of the measure of the quantity on which it operates (Schwartz,1988)
f. The comprehension of the natural number as operator requires a process of abstraction (Steffe,1994).

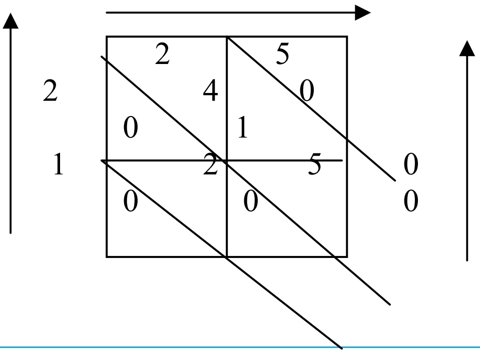
According to many researchers (Piaget,1973a-Gelman,1978-Vergnaud,1983), children can construct the concept of the natural number sufficiently, if they are engaged in activities concerning additive as well as multiplicative structures. Although there are many researches that refer to the way that kindergarten children build the concept of number approaching it from its cardinal and ordinal nature (Fuson,1988), kindergarten children's engagement with activities concerning the number as operator has a special research interest.

Additionally, it has been noticed that even young children have the ability to deal with difficult ideas, if these ideas are presented in ways that attract their interest and have a meaning to them. Especially, many educators have recognized the predominant role of play to children's cognitive evolution at this age (Kamii & de Clark, 1985 - Van Oers, 1996-Renshaw 1992- Gelman, 1978).

According to the historians of mathematics and other researchers (Ifrah, 1985- Struick 1982- Exarhakos, 1988- Boufi, 1995- Toumasis, 1994) we observe that primitive people used the natural number as operator to solve practical problems of everyday life or problems that were related to professional subjects or other subjects of social nature. It is worthwhile to mention an example for the use of number as operator. Ifrah has mentioned a technique of the multiplication of ancient Egyptians, who without knowing to multiply or divide, except with the number 2, they made doublings successively, namely series of multiplications with 2, where 2 had the role of operator, namely of the equal quantities that had to be added. E.g for the multiplication 16 x 12, they wrote 2 vertical columns as following:

1	12
2	24
4	48
8	96
16	192

We observe that they double successively each one of the numbers till it appears at the left column the number 16. Then at the right column it appears the product of 16 x 12, 192. The number 16 is the operator, but because it was not easy to execute the operation, the operator was analyzed, creating a new one, the number 2, which influences on both columns bringing transformations. Later on, from the 6th century and forward, mathematicians from India used to use a quite peculiar way for their multiplications. This way was transmitted to Arabs and from them to Europeans and it was named "multiplication based on the square". The mathematicians from India formed a rectangular table with as many vertical columns as the numerical digits of the multiplicand and as many horizontal columns as the numerical digits of the multiplier. E.g. for the multiplication of 12 x 25:



On the top of the table and from the left to the right they used to write the numerical digits of the multiplicand (25). At the left side of the table they used to write the digits of the multiplier (12) beginning from the bottom to the top. Following the used to divide the rectangular made by the horizontal and the vertical columns, with the table diagonals, as it appears at the shape. Next they used to multiply. Afterwards, at each square they wrote the product of the two numbers, which were at the beginning of the line and on the top of the columns respectively (this product it is implied that was less than 100). Then they wrote the number of the decades at the bottom left triangle and the number of the units at the right triangle on the top (at the triangles which were formed by diagonals). If the result of the product didn't have one of the two orders of the numeration, they put 0 at the respective place. Following they added the numbers that was included at the diagonals starting from the upper right side of the rectangular and they wrote the sum out of the rectangular at the continuation of the diagonal. They read the result of the product beginning from the left bottom, continuing to the upper right side of the rectangular (Ifrah, 1985). When it was for products of smaller numbers, it was relatively simple. When it was for products with big numbers, then these tables were useful and effective, of course not for the everyday people, but for the experts of this period.

In both examples we can observe that even at this period of time, that knowledge was not advanced, the number as operator was used, either to double a quantity, or by being analysed to a smaller number that used to bring successive transformations to the original quantity, aiming to solve problems of everyday needs.

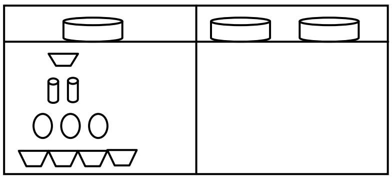
In Greece, children, at early childhood, work with activities oriented to the additive structures and they start to approach problems concerning the multiplicative structures in a greater level. The activities, that are going to be discussed in this workshop, are part of a broader research program, which aims to the study of changes that occur to kindergarten's children conception about natural number, when activities referring to the number as operator are proposed. According to Schwartz(1988) the multiplication and division are referent transforming compositions (they compose two quantities to produce a third quantity that is like neither of the two original quantities). Referent transforming compositions force us to distinguish between two rather different kinds of quantity, extensive quantity (E) and intensive quantity (I). The statement of an intensive quantity is a statement of a relationship between quantities (e.g. 5 candies per bag). The related quantities are extensive quantities. (Schwartz, 1988). The activities used were of the type I x E where the result or the multiplier was inquired and activities in which the natural number functions as an operator (S x E) **.**

Workshop's organization

At the workshop we discussed with the participants representative samples of activities that they were given to kindergarten children. At the first phase we discussed the sample space of the solutions on these activities. At the second phase we studied the children's strategies from the research program in order to compare them with the previous solutions. At the third phase we discussed the following questions: a) How children's participation can represent their way of thinking numbers? b) What characteristics of these solutions can be interpreted as a primitive comprehension of the concept of the natural number as operator? The participants separated in groups in order to work out the specific activities and to discuss their thoughts. Afterwards all the participants were involved in a general discussion concerning the topic of the research.

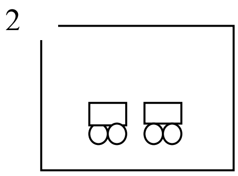
Results and discussion

During the working group we discussed three activities: «A recipe for one / two cakes»,«Sticks and Boxes» and the «Magic box». The objective of the first activity is to approach the concept of number as operator by doubling. The children are asked to make the recipe for two cakes given a recipe for one cake.

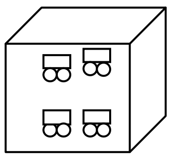


The second activity was a problem of the type: ? X a=b. The scenario of the activity is the following: A model of a stick and a box is given. The children have to be separated in groups and to collaborate in order to take a decision about the number of sticks they will use making the final construction of the given box. When they construct the box, they compare their own box with the given box and they decide if it is the same construction or it is necessary to do changes. The children are asked to answer questions like: How many sticks / pairs do you need to make this box?

Through the third activity the children can approach the concept of number as operator by transforming the number. There is a «magic» box (figure 2). This “magic box” has the property to increase the quantity of things that there are inside it. There is a card (figure1) with numbers that signify the quantitative changes of the things included in the box.



(Fig. 1)



(Fig. 2)

Whenever the number on the card changes (the number shows the transformation of the quantity), the children have to think and to find how many pairs of things are in the box. They also find how many items of things are, if they are counted one by one. The children are separated in groups. The group’s decision is announced by writing the number on a card or by the group’s leader oral announcement. For every correct answer of the team, the leader takes a bonus- card. At the end of the game the team’ s leaders count the bonus – cards and proclaims the winner team.

Then we present the discussion developing among the participants for the first activity. In an analogous manner we discussed the other two activities.

The early childhood children, who participated in our research, gave the following solutions at the first activity:

A) The children used objects and they said: “One and another one glass of milk, two

and another two vanillas, three and another three eggs, four and another four glasses of flour".
B) The children duplicated the quantity of ingredients of the recipe for one cake in order to calculate the quantity of ingredients for two cakes. They used objects and they said: "Two times from one glass of milk, two times from two vanillas, two times from three eggs, two times from four glasses of flour".
C) The children made mental duplicating of the quantity of ingredients of the recipe for one cake in order to calculate the quantity of ingredients for two cakes and they told the result of the multiplication. They used objects and they said and wrote: 2 (glasses of milk), 4 (vanillas), 6 (Eggs), 8 (glasses of flour).

The participants gave the following solutions:

a) (a+2b+3c+4b) + (a+2b+3c+4b), namely: one glass of milk, two vanillas, three eggs, four glasses of flour **and** one glass of milk, two vanillas, three eggs, four glasses of flour.
b) a+ a, 2b+2b, 3c+ 3c, 4d+4d, namely: One and another one glass of milk, two and another two vanillas, three and another three eggs, four and another four glasses of flour.
c) (a·2) + (2b· 2) + (3c· 2) + (4d·2), namely : Two times from one glass of milk, two times from two vanillas, two times from three eggs, two times from four glasses of flour.

The basic conclusion from our discussion was the difficulty to characterize the children's solutions as a primitive comprehension of the concept of the natural number as operator. The participants proposed and discussed different criteria about the above issue. All the comments were focused on the way that children express their thinking. The strategy that they use when they count, the use of the word "times" and the way that they wrote their thinking can allow the interpretation of a solution as an indication of the concept of number as operator. However, these elements can not be studied separately, but in coordination.

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