A two-layered blood flow through an overlapping constriction with permeable wall

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Abstract

The present work concerns with the effects of the permeability of the wall through an overlapping constriction in an artery assuming that the flowing blood is represented by a two-fluid model. The expressions for the blood flow characteristics, the impedance, the wall shear stress distribution in the stenotic region and the shearing stress at the stenosis throat have been derived. Results for the effects of permeability as well as of the peripheral layer on these blood flow characteristics are shown graphically and discussed briefly.

Key words: Permeable, Darcy number, slip parameter, impedance, shear stress, stenosis throat.

INTRODUCTION

There has been growing interest in studying blood rheology and blood flow. The frequently occurring cardiovascular disease, arteriosclerosis or stenosis, responsible for many of the diseases e.g., myocardial infarction, cerebral strokes, angina pectoris, etc., is the unnatural and abnormal growth that develops at various locations of the cardiovascular system under diseased conditions. Although, the etiology of the initiation of the disease (stenosis) is not well understood, its subsequent and severe growth on the artery wall results in serious circulatory disorders, but it is well established that once the constriction has developed, it brings about the significant changes in the flow field i.e., pressure distribution, wall shear stress, impedance, etc.. With the knowledge that the haemodynamic factors play an important in the genesis and the proliferation of arteriosclerosis, since the first investigation of Mann et al. (1938), a large number of researcher have addressed the stenotic development problems under various flow situations including Young (1968), Young and Tsai (1973), Caro et al. (1978), Shukla et al. (1980), Liu et al. (2004), Srivastava and coworkers (2010, 2012), Mishra et al. (2006), Ponalagusamy (2007), Layek et al. (2009), Tzirtzilakis (2008), Mandal et al. (2007), Politis et al. (2008), Medhavi (2011, 2012a,b), and many others.

The flowing blood has been represented by a Newtonian, non-Newtonian, single or double-layered fluid by the investigators in the literature while discussing the flow through stenoses. It is well known that blood may be represented by a single-layered model in large vessel, however, the flow through the small arteries is known to be a two-layered. Bugliarello and Sevilla (1970) and Cokelet (1972) have shown experimentally that for blood flowing through small vessels, there is cell-free plasma (Newtonian viscous fluid) layer and a core region of suspension of all the erythrocytes. Srivastava (2007) concluded that the significance of the peripheral layer increases with decreasing blood vessel diameter.

The plasma membrane is a thin, elastic membrane around the cell which usually allows the movement of small irons and molecules of various substances through it. This nature of plasma membrane is termed as permeability. In addition, the endothelial walls are known to be highly permeable with ultra microscopic pores through which filtration occur. Cholesterol is believed to increase the permeability of the arterial wall. Such increase in permeability results from dilated, damaged or inflamed arterial walls.

In view of the discussion given above, the research reported here is therefore devoted to discuss the two-layered blood flow through an overlapping stenosis in an artery with permeable wall. The mathematical model considers the flowing blood as a two-layered Newtonian fluid, consisting of a core region (central layer) of suspension of all the erythrocytes assumed to be a Newtonian fluid, the viscosity of which may vary depending on the flow conditions and a peripheral region (outer layer) of another Newtonian fluid (plasma) of constant viscosity, in an artery with permeable wall.

FORMULATION OF THE PROBLEM

Consider the axisymmetric flow of blood in a uniform rigid circular artery of radius R with an axisymmetric overlapping stenosis. Blood is assumed to be represented by a two-layered model consisting of a central layer of suspension of all the erythrocytes, assumed to be a Newtonian fluid of radius R_1 and a peripheral layer of plasma (a Newtonian viscous fluid of constant viscosity) of thickness $(R-R_1)$. The stenosis geometry and the shape of the central layer, assumed to be manifested in the arterial segment, are described (Srivastava and Saxena, 1994; Layek, et al., 2009) in Figs. 1and 2, respectively, as

$$\frac{(R(z),R_1(z))}{R_0} = (1,\beta) - \frac{3}{2}\frac{(\delta,\delta_1)}{R_0 L_0^4}\left[11(z-d)L_0^3 - 47(z-d)^2 L_0^2 + 72(z-d)^3 L_0 - 36(z-d)^4\right]; \quad d \le z \le d + L_0,$$

$$= (1,\beta); \quad \text{otherwise}, \tag{1}$$

where $R \cong R(z)$ and R_0 are the radius of the tube with and without constriction, respectively; $R_1(z)$ is the radius of the central layer; L is the tube length, L_0 is the stenosis length and d indicates the location of the stenosis, β is the ratio of the central core radius to the tube radius in the unobstructed region and (δ, δ_1) are the maximum height of the stenosis and bulging of the interface at two locations in the stenotic region at $z = d+L_0/6$ and $z = d+5L_0/6$. The stenosis height located at $z = d+L_0/2$, called critical height, is $3\delta/4$.

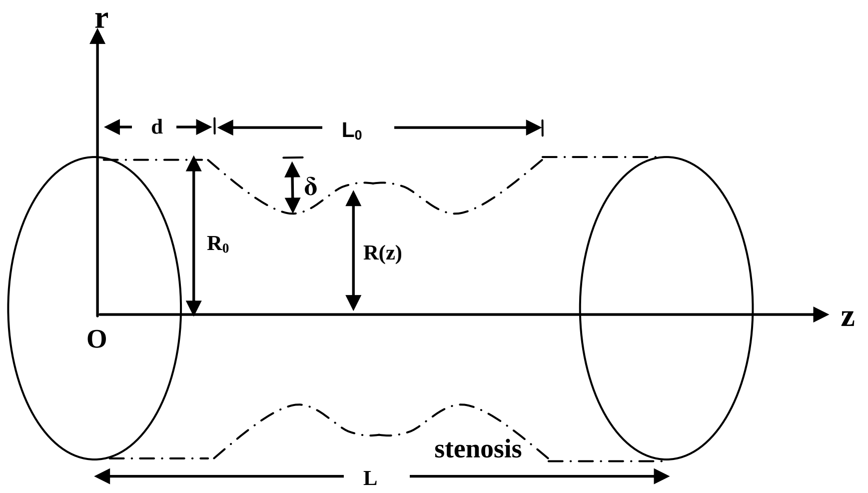

Fig.1a Flow geometry of an arterial overlapping stenosis with permeable wall.

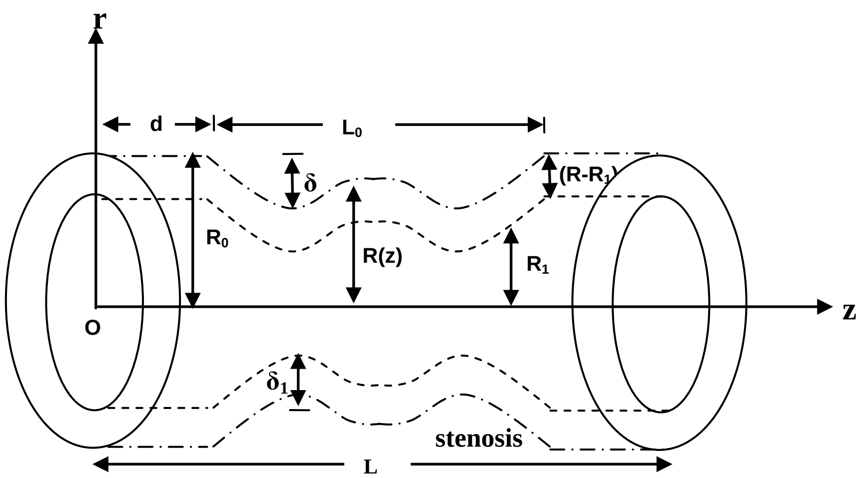

Fig.1b Flow geometry of an arterial overlapping stenosis with peripheral layer.

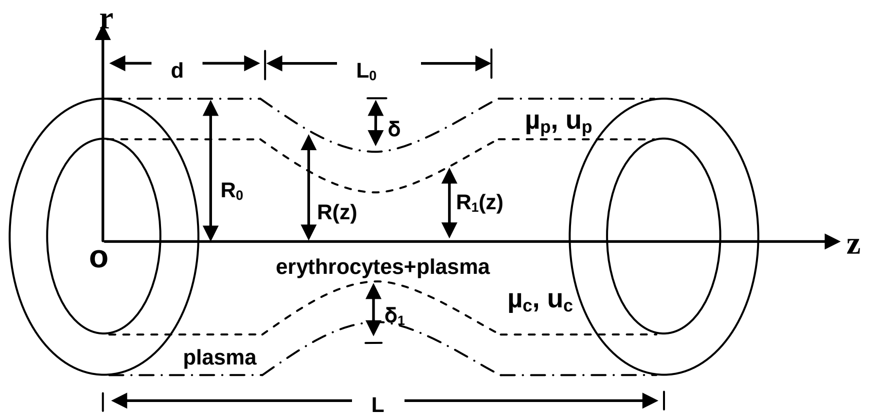

Fig.2 The shape of the central layer

The equations describing the laminar, steady, one-dimensional flow in the case of a mild stenosis $(\delta << R_0)$ are expressed (Young, 1968; Sharan and Popel, 2001) as

$$\frac{dp}{dz}=\frac{\mu_p}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})\,u_p\,, \qquad R_1(z)\le r\le R(z), \tag{2}$$

$$\frac{dp}{dz}=\frac{\mu_c}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u_c\,, \qquad 0\le r\le R_1(z), \tag{3}$$

where (u_p, μ_p) and (u_c, μ_c) are (velocity, viscosity) of fluid in the peripheral layer $(R_1(z)\le r\le R(z))$ and central layer $(0\le r\le R_1(z))$, respectively; p is the pressure and (r, z) are (radial, axial) coordinates in the two-dimensional cylindrical polar coordinate system.

The conditions that are satisfied at the artery wall and the interface for the present study may now be stated (Beavers and Joseph, 1967; Srivastava et al., 2012) as

$$\frac{\partial u_c}{\partial r}=0 \quad \text{at } r=0 \tag{4}$$

$$u_p=u_c \text{ and } \mu_p\frac{\partial u_p}{\partial r}=\mu_c\frac{\partial u_c}{\partial r} \quad \text{at } r=R_1(z), \tag{5}$$

$$u_p=u_B \quad \text{and} \quad \frac{\partial u_p}{\partial r}=\frac{\alpha}{\sqrt{k}}\left(u_B-u_{porous}\right) \quad \text{at } r=R(z), \tag{6}$$

where $u_{porous}=-\frac{k}{\mu_p}\frac{dp}{dz}$, u_{porous} is the velocity in the permeable boundary, u_B is the slip velocity, μ_p is the plasma viscosity (fluid viscosity in the peripheral layer), k is Darcy number and α (called the slip parameter) is a dimensionless quantity depending on the material parameters which characterize the structure of the permeable material within the boundary region.

ANALYSIS

The straight forward integration of Eqns. (2) and (3), subject to the boundary conditions (4), (5) and (6), yields the expressions for velocity, u_p and u_c as

$$u_p=-\frac{R_0^2}{4\mu_p}\frac{dp}{dz}\{(R/R_0)^2-(r/R_0)^2-2(R/R_0)(\sqrt{k}/\alpha R_0)+4k/R_0^2\}, \tag{7}$$

$$u_c=-\frac{R_0^2}{4\mu_p}\frac{dp}{dz}\{(R/R_0)^2-\mu(r/R_0)^2-(1-\mu)(R_1/R_0)^2+4k/R_0^2-2(R/R_0)(\sqrt{k}/\alpha R_0)\}, \tag{8}$$

with $\mu=\mu_p/\mu_c$.

The volumetric flow rate, Q is now calculated as

$$Q = 2\pi\left\{\int_0^{R_1} r\,u_c\,dr + \int_{R_1}^{R} r\,u_p\,dr\right\}$$
$$= -\frac{\pi R_0^4}{8\,\mu_p}\frac{dp}{dz}\{(R/R_0)^4 - (1-\mu)(R_1/R_0)^2 + 8k(R/R_0)^2/R_0^2 - 4\sqrt{k}\,(R/R_0)^3/\alpha R_0\}. \tag{9}$$

Following the argument that the total flux is equal to the sum of the fluxes across the two regions (central and peripheral), one derives the relations (Srivastava and Saxena, 1994): $R_1 = \beta R$ and $\delta_1 = \alpha\delta$ $(0 \le \beta \le 1)$. An application of these relations into the Eqn. (9), yields

$$\frac{dp}{dz} = -\frac{8\,\mu_p\,Q}{\pi\,R_0^4}\,F(z), \tag{10}$$

where $F(z) = 1/\{(1-(1-\mu)\beta^4)(R/R_0)^4 + 8k(R/R_0)^2/R_0^2 - 4\sqrt{k}\,(R/R_0)^3/\alpha R_0\}$.

The pressure drop, $\Delta p\,(= p \text{ at } z = 0, -p \text{ at } z = L)$ across the stenosis in the tube of length, L is obtained as:

$$\Delta p = \int_0^L\left(-\frac{dp}{dz}\right)dz$$
$$= \frac{8\,\mu_p\,Q}{\pi\,R_0^4}\left\{\int_0^d [F(z)]_{R/R_0=1}dz + \int_d^{d+L_0} F(z)\,dz + \int_{d+L_0}^{L}[F(z)]_{R/R_0=1}dz\right\}. \tag{11}$$

The analytical evaluation of the first and the third integrals on the right hand side of Eqn. (11) are the straight forward, whereas the evaluation of the second integral seems to be a difficult task thus shall be evaluated numerically. One now derives the expressions for the impedance (flow resistance), λ, the wall shear stress distribution in stenotic region, τ_w, the shear stress at the stenosis throats, τ_s and the shear stress at the stenosis critical height, τ_c using the definitions from the published literature (Young, 1968, Srivastava and Saxena, 1994), in their non-dimensional form as

$$\lambda = \mu\left\{\frac{(1 - L_0/L)\eta_1}{\eta} + \frac{\eta_1 L_0}{2\pi\ L}\int_d^{d+L_0} F(z)\,dz\right\} \tag{12}$$

$$\tau_w = \frac{\mu\eta_1}{(1-(1-\mu)\beta^4)(R/R_0)^3 + 8k(R/R_0)/R_0^2 - 4\sqrt{k}\,(R/R_0)^2/\alpha R_0}, \tag{13}$$

$$\tau_s = [\tau_w]_{R/R_0 = 1-5\delta/4R_0}, \tag{14}$$

$$\tau_c = [\tau_w]_{R/R_0=1-3\delta/4R_0}, \tag{15}$$

where $\eta_1 = 1 + 8k/R_0^2 - 4\sqrt{k}/\alpha R_0$, $\eta = 1-(1-\mu)\beta^4 + 8k/R_0^2 - 4\sqrt{k}/\alpha R_0$, $\lambda = \bar{\lambda}/\lambda_0$, $(\tau_w, \tau_s, \tau_c) = (\bar{\tau}_w, \bar{\tau}_s, \bar{\tau}_c)/\tau_0$.

Also $\lambda_0 = 8\mu_c L/\eta_1 \pi R_0^4$ and $\tau_0 = 4\mu_c Q/\eta_1 \pi R_0^3$ are the flow resistance and shear stress, respectively for a single-layered Newtonian fluid in a normal artery (no stenosis) with permeable wall and $(\bar{\lambda}, \bar{\tau}_w, \bar{\tau}_s, \bar{\tau}_c)$ are respectively, (the impedance, the wall shear stress, the shearing stress at stenosis throats, the shear stress at the stenosis critical height) in their non-dimensional form obtained from the definitions (Young, 1968): $\bar{\lambda} = \Delta p/Q$, $\bar{\tau}_W = -(R/2)\,dp/dz$, $\bar{\tau}_s = [\bar{\tau}_w]_{R/R_0=1-5\delta/4R_0}$ and $\bar{\tau}_c = [\bar{\tau}_w]_{R/R_0=1-3\delta/4R_0}$.

In the absence of the permeability in the artery wall (i.e., k = 0), the results obtained in the Eqns. (12) - (15) take the form

$$\lambda = \mu\left\{\frac{(1-L_0/L)\,\eta_1}{\eta} + \frac{\eta_1 L_0}{2\pi\ L}\int_d^{d+L_0} F_1(z)\,dz\right\} \tag{16}$$

$$\tau_w = \frac{\mu\eta_1}{(1-(1-\mu)\beta^4)\,(R/R_0)^3}, \tag{17}$$

$$\tau_s = \frac{\mu\eta_1}{(1-(1-\mu)\beta^4)\,(1-5\delta/4R_0)^3}, \tag{18}$$

$$\tau_c = \frac{\mu\eta_1}{[1-(1-\mu)\beta^4]\,(1-3\delta/4R_0)^3}, \tag{19}$$

where $F_1(z) = 1/\{1-(1-\mu)\beta^4)\,(R/R_0)^4\}$, which correspond to the results derived in the case of two-layered model analysis of a Newtonian fluid. When the viscosity of the fluid in the peripheral region is the same as the viscosity of the fluid in the core region (i.e., $\mu = 1$), one derives the results for a single-layered analysis of a Newtonian fluid in the presence of the permeability in the artery wall as

$$\lambda = \mu\left\{\frac{(1-L_0/L)\eta_1}{\eta} + \frac{\eta_1 L_0}{2\pi\ L}\int_d^{d+L_0} F_2(z)\,dz\right\} \tag{20}$$

$$\tau_w = \frac{\mu\eta_1}{(R/R_0)^3 + 8k(R/R_0)/R_0^2 - 4\sqrt{k}\,(R/R_0)^2/\alpha R_0}, \tag{21}$$

$$\tau_w = \frac{\mu\eta_1}{\left(1-5\,\delta/4R_0\right)^3 + 8k\left(1-5\delta/4R_0\right)/R_0^2 - 4\sqrt{k}\left(1-5\,\delta/4R_0\right)^2/\alpha R_0}, \qquad (22)$$

$$\tau_c = \frac{\mu\eta_1}{\left(1-3\,\delta/4R_0\right)^3 + 8k\left(1-3\delta/4R_0\right)/R_0^2 - 4\sqrt{k}\left(1-3\,\delta/4R_0\right)^2/\alpha R_0}, \qquad (23)$$

It is further to note that the results corresponding to that of Young (1968) for an overlapping stenosis in the case of a Newtonian fluid with impermeable wall may be derived from Eqns. (12) - (15) by setting k = 0 and μ = 1.

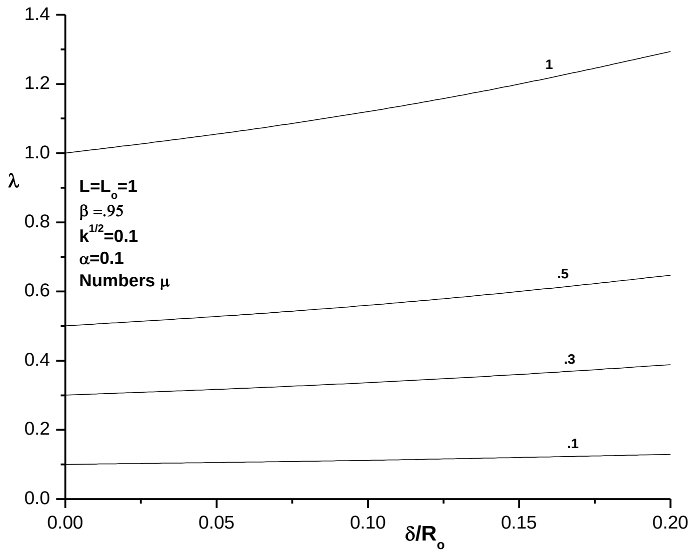

Fig.3 Impedance, λ vs stenosis height, δ/R$_o$ for different μ.

NUMERICAL RESULTS AND DISCUSSION

Blood flow characteristics in arteries can be altered significantly by arterial disease, stenosis. To discuss the results of the study quantitatively, computer codes are developed to evaluate the analytical result for flow resistance, λ, the wall shear stress, τ_w, and shear stress at the stenosis throat, τ_s obtained above in Equations (12) - (14) for various parameter values and some of the critical results are displaced graphically in Figures (3-17). The various parameters are selected (Young, 1968; Beavers and Joseph, 1967; Srivastava et al., 2012) as: L_0 (cm) = 1; L (cm) = 1, 2, 5, 10; α = 0.1, 0.2, 0.3, 0.5; $\sqrt{k}$ = 0, 0.1, 0.2, 0.3, 0.4, 0.5; β = 1, 0.95, 0.90; μ =1, 0.5, 0.3, 0.1; and δ/R_0 = 0, 0.5, 0.10, 0.45, 0.20; etc.. It is worth mentioning here that present study corresponds to impermeable artery case, to single-layered model study, and to no stenosis case for parameter values $\sqrt{k}$ (here and after called Darcy number) = 0; $\beta = 1$ or $\mu = 1$, and $\delta/R_0 = 0$; respectively.

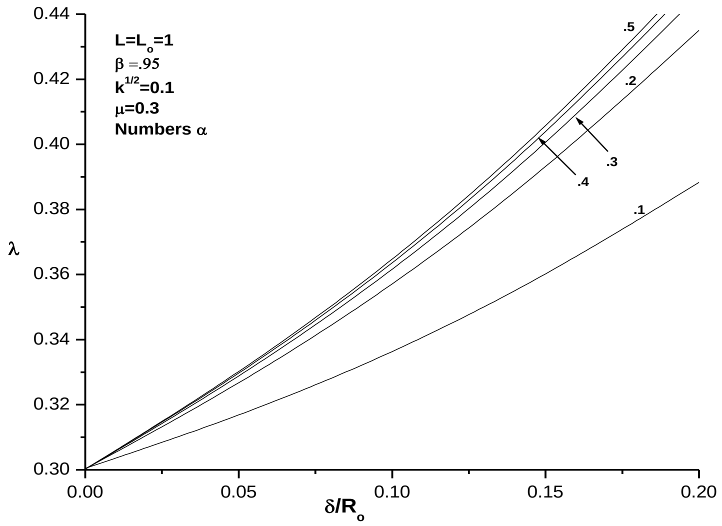

Fig.4 Impedance, λ vs stenosis height, δ/R_o for different α.

The flow resistance λ, increases with the stenosis height, δ/R_0, for any given set of parameters. At any given stenosis height, δ/R_0, λ decreases with the peripheral layer viscosity, μ from its maximal magnitude obtained in a single-layered study (i.e., $\mu=1$ or $\beta=1$, Fig.1a).

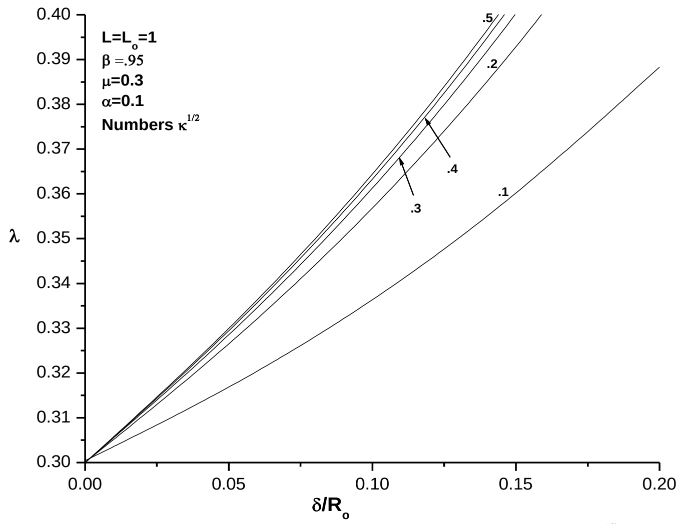

Fig.5 Impedance, λ vs stenosis height, δ/R_o for different $\kappa^{1/2}$.

One observes that at any given stenosis height, δ/R_0, the impedance, λ increases with the slip parameter, α (Fig.4).

The blood flow characteristic, λ increases with the Darcy number, $\sqrt{k}$ at any given stenosis height, δ/R_0 (Fig.5).

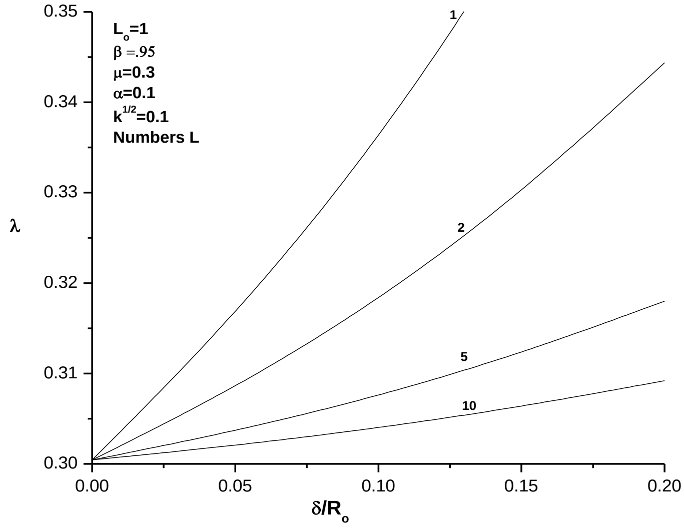

Fig.6 Impedance, λ vs stenosis height, δ/R_o for different L.

The impedance, λ decreases with increasing tube length L which interns implies that λ increases with increasing value of L_0/L (stenosis length, Fig.6).

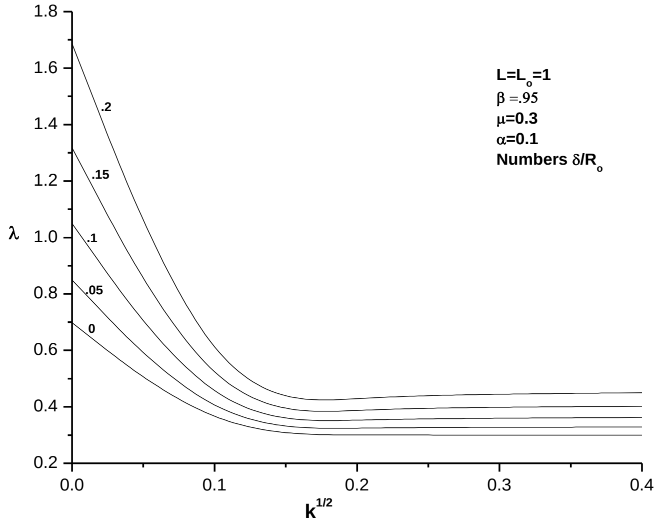

Fig.7 Impedance, λ versus Darcy number, $k^{1/2}$ for different δ/R_o.

One observes that the flow resistance, λ decreases rapidly with increasing value of the Darcy number, $\sqrt{k}$ from its maximal magnitude at $\sqrt{k}=0$ (impermeable wall) in the range $0 \le \sqrt{k} \le 0.15$ and afterwards assumes an asymptotic value with increasing values of the Darcy number, $\sqrt{k}$ (Fig.7).

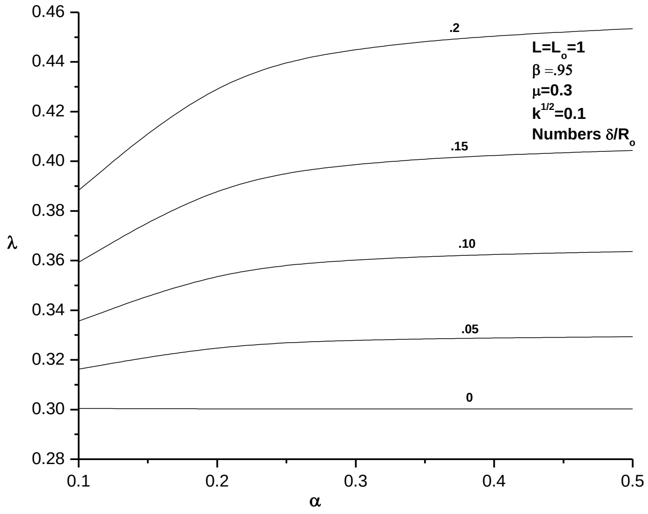

Fig.8 Impedance, λ versus slip parameter, α for different δ/R$_o$.

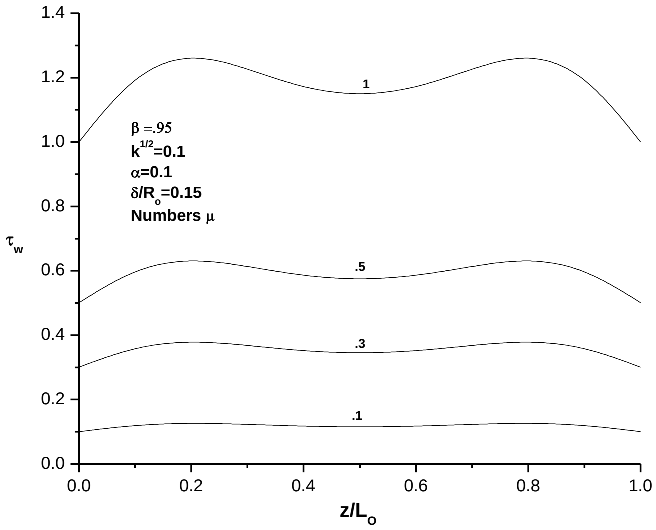

Fig.9 Wall shear stress τ_w in the stenotic region for different μ .

We notice that the blood flow characteristic, λ increases with the slip parameter, α from its minimal magnitude at $\alpha = 0.1$ and approaches to an asymptotic magnitude when α increases from 0.2 (Fig.8).

The wall shear in the stenotic region, τ_w increases from its approached value at $z/L_0 = 0$ to its peak value at $z/L_0 = 0.5$ and then decreases from its peak value to its approached value at the end point of the constriction profile at $z/L_0 = 1$ for any given set of parameters (Figs. 9-12).

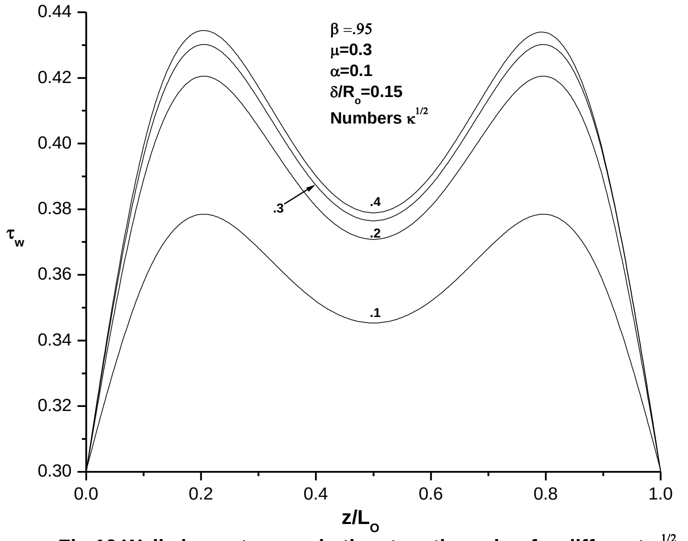

Fig.10 Wall shear stress τ_w in the stenotic region for different $\kappa^{1/2}$.

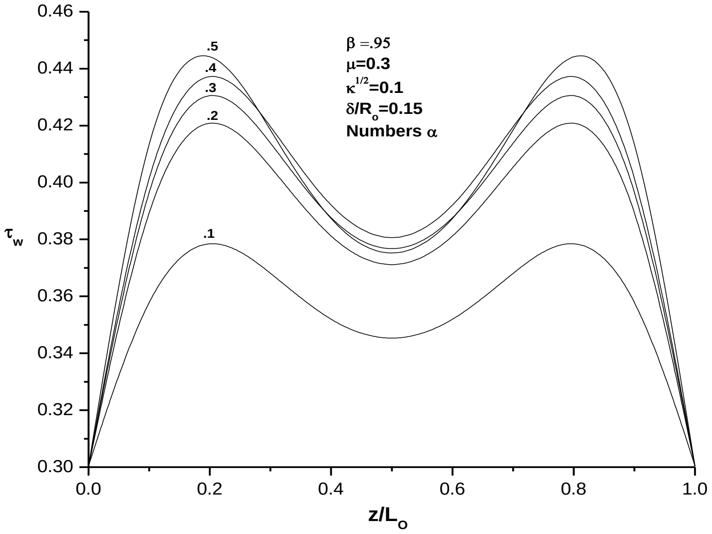

Fig.11 Wall shear stress τ_w in the stenotic region for different α.

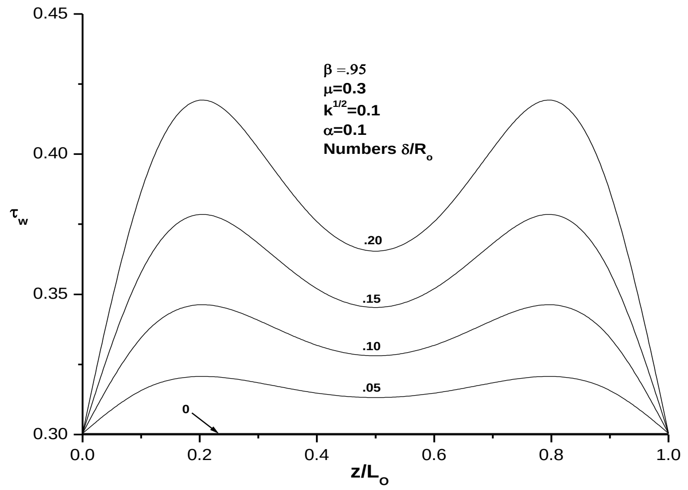

Fig.12 Wall shear stress in the stenotic region for different δ/R_o.

The blood flow

characteristic, τ_s possesses characteristics similar to that of the flow resistance, with respect to any parameter (figs. 13-17). Numerical results reveal that the variations of the shear stress, τ_s are similar to that of the impedance (flow resistances), λ with respect to any parameter.

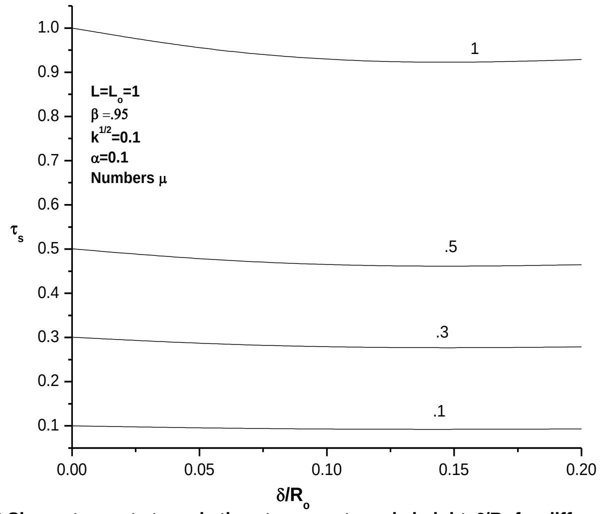

Fig.13 Shear stress at stenosis throats, τ_s vs stenosis height, δ/R_o for different μ .

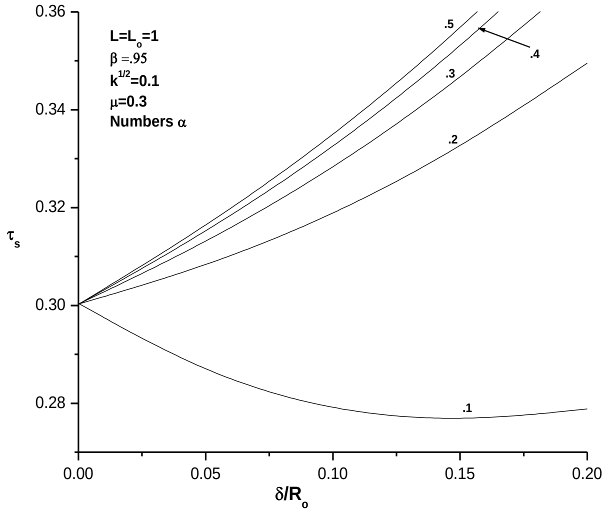

Fig.14 Shear stress at stenosis throats, τ_s vs stenosis height, δ/R_o for different α.

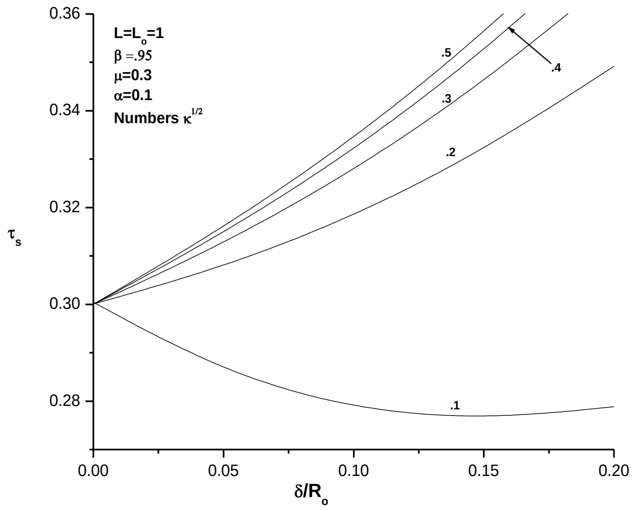

Fig.15 Shear stress at stenosis throats, τ_s vs stenosis height, δ/R_o for different $\kappa^{1/2}$.

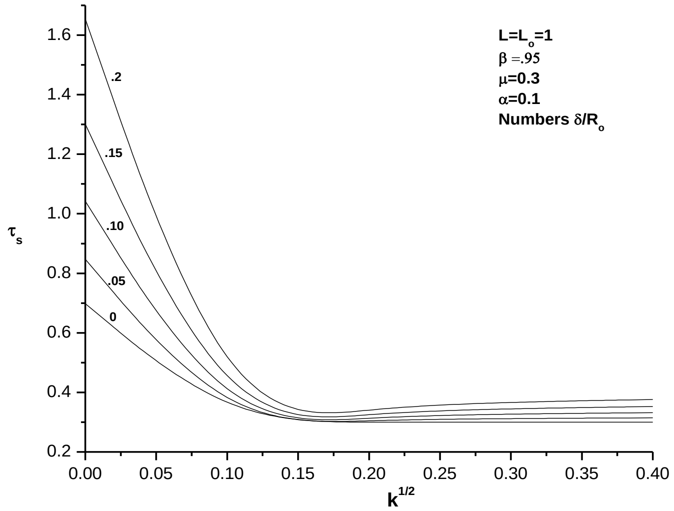

Fig.16 Shear stress at stenosis throats, τ_s versus Darcy number, $k^{1/2}$for different δ/R_o.

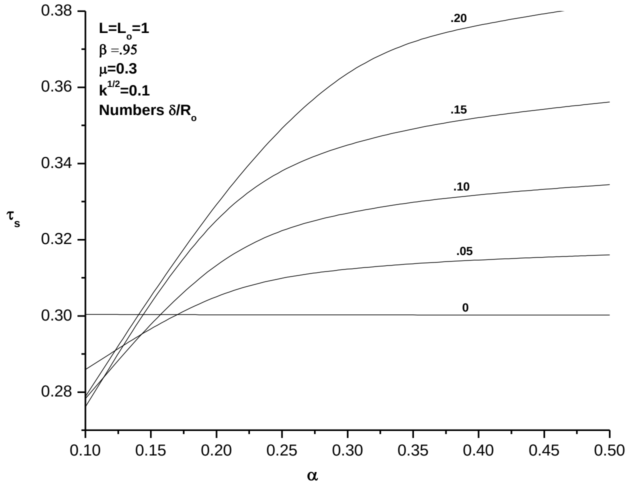

Fig.17 Shear stress at stenosis throats, τ_s versus slip parameter, α for different δ/R_o.

CONCLUSIONS

To observe the effects of the permeability of the artery wall and the peripheral layer on blood flow characteristics due to the presence of a stenosis, a two-fluid blood flow of Newtonian fluid through an axisymmetric stenosis in an artery with permeable wall has been studied. The study enables one to observe the simultaneous effects of the wall permeability and the peripheral layer on blood flow characteristics due to the presence of a stenosis. For any given set of parameters, the blood flow characteristics (impedance, wall shear stress, etc.) assume lower magnitude in two-fluid model than its corresponding value in one-fluid analysis. The impedance decreases with increasing Darcy number from its maximal magnitude in the case of impermeable wall (i.e., at zero Darcy number). It is therefore concluded that the existence of permeability in the artery wall and the presence of the peripheral layer in the artery help the functioning of the diseased artery.

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