PERISTALTIC TRANSPORT IN A CIRCULAR CYLINDRICAL TUBE WITH PERMEABLE WALL

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ABSTRACT
Peristaltic pumping of a particle-fluid suspension in a catheterized circular tube has been investigated. The coupled differential equations for both the fluid and the particle phases have been solved and the expressions for the flow rate, pressure drop, friction forces at the tube and the catheter wall have been derived. It is found that the pressure drop, Δp decreases with increasing flow rate, Q for any given value of the slip parameter, α Darcy number, $K^{1/2}$ amplitude ratio, $\emptyset$ particle concentration and catheter size. Also for any given flow rate and the catheter size, pressure drop decreases with the particle concentration, and assumes significantly higher magnitude in a catheterized tube than its corresponding value in uncatheterized tube. The friction forces, F (at tube as well as the catheter wall) possess characteristics similar to the pressure drop (an opposite characteristics to the pressure rise) with respect to any parameter. The friction force at the tube wall is found to be significantly higher in magnitude than the corresponding friction force at the catheter wall.

Keywords: Peristaltic, flow-rate, pressure drop, friction force.

INTRODUCTION

Radhakrishnamacharya (1982) studied long wavelength approximation to peristaltic motion of a power law fluid. The inertia – free peristaltic flow with long wavelength analysis was given by Shapiro et al. (1969). The early developments on the mathematical modeling and experimental fluid mechanics of peristaltic flow were given in a comprehensive review by Jaffrin and Shapiro (1971). Beavers and Joseph (1967) developed boundary conditions at a naturally permeable wall. However, the rheological properties of the fluids can affect these characteristics .Flow through a porous medium have several practical applications especially in geophysical fluid dynamics. Examples of natural porous media are beach sand, sandstone, limestone, the human lung, bile duct, gall bladder with stones in small blood vessels. El Shehawey and Husseny (1999) and El Shehawey et al. (2000) studied the peristaltic flow of a Newtonian fluid thorough a porous medium.
Caro, Pedley, Schroter and Seed (1978) studied the mechanics of the Circulation. Moreover, most of the physiological fluids are known to be non-Newtonian. Very little attention has also been paid to the peristaltic flows of non-Newtonian fluids. Shukla and Gupta (1982) was studied the peristaltic transport of a power- law fluid with variable consistency. Peristaltic transport of non - Newtonian fluids with the application to the vas deferens and small intestine was studied by Srivastava. Consequently, peristaltic transport of power law fluid has been discussed by Srivastava and Srivastava (1988) with the application to the ducts deferens of the

reproductive tract. The non - Newtonian peristaltic flow using a constitutive equation for a second order fluid has been investigated by Siddiqui et al. (1991) for a planar channel and by Siddiqui and Schwarz (1994) for an asymmetric tube. They have performed a perturbation analysis with a wave number, including curvature and inertia effects .In further investigation many authors have used one of the simplification is that they have assumed blood to be a suspension of spherical rigid particles (red cells), this suspension of spherical rigid particles will give rise to couple stresses in a fluid. No effort in literature has been made to understand the peristaltic flow of a Newtonian Fluid in a channel with permeable walls.
The aim of the present investigation is to study the peristaltic transport in a circular cylindrical tube with permeable wall.

FORMULATION OF THE PROBLEM

Consider the flow of a Newtonian fluid through a circular cylindrical tube with permeable wall. The tube wall is assumed to be flexible and the flow is induced by a sinusoidal wave travelling down its wall. The geometry of the wall surface is described (Fig. 1) as

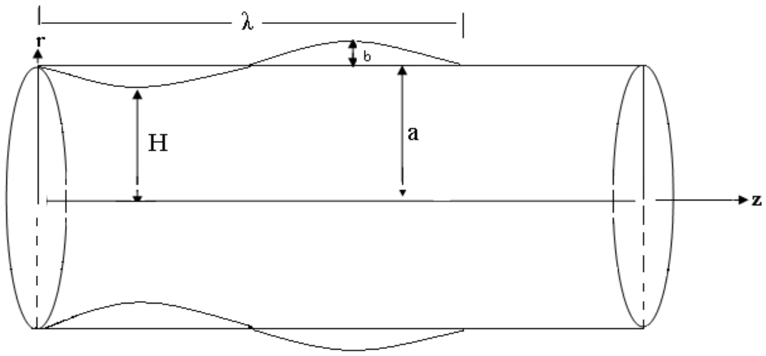

Fig. 1 Geometrical representation of peristaltic waves in a circular cylindrical tube with permeable wall.

$$H(z,t) = a + b\ \mathrm{Sin}\frac{2\pi}{\lambda}(z - ct) \tag{1}$$

Where 'a' is the radius of the tube, 'b' is the amplitude of the peristaltic wave, 'λ' is the wavelength, 'c' is the wave propagation speed, 't' is the time and 'z' is the axial coordinate.
The equations governing the flow of a Newtonian fluid are the linear momentum and the conservation of man and are stated as

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial z} + V\frac{\partial u}{\partial R}\right) = -\frac{\partial P}{\partial z} + \mu\left\{\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial}{\partial R}\right) + \frac{\partial^2}{\partial z^2}\right\}U \tag{2}$$

$$\rho\left(\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial z} + V\frac{\partial V}{\partial R}\right) = -\frac{\partial P}{\partial R} + \mu\left\{\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial}{\partial R}\right) - \frac{1}{R^2} + \frac{\partial^2}{\partial z^2}\right\}V \tag{3}$$

$$\frac{1}{R}\frac{\partial}{\partial R}(RV) + \frac{\partial u}{\partial z} = 0 \tag{4}$$

Where (U, V) are the (axial, radial) components of velocity of the fluid, (ρ, μ) are the (density, viscosity) of the fluid, 'P' is the pressure and (z, R) are two-dimensional cylindrical polor coordinate.
The flow induced by the peristaltic wave is unsteady in the fixed frame of reference (Z, R). However if one chooses a moving frame reference (z, r) with the speed of the peristaltic wave, 'c' in z-direction, two flow can be treated as steady. The transformation from a fixed to the moving frame of reference in coordinates are z= Z – ct and r=R, and in the velocity components are u= U – c and v = V. For the pressure is p(z, r) = P(Z, R, t) and the dimensional equation of the tube wall in the moving frame is

$$h'(z) = a + b\,\mathrm{Sin}\frac{2\pi}{\lambda}z \qquad (5)$$

The equations of motion Eqns. (2) – (4) governing the flow in the moving frame of reference assume the form

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial z} + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}\right\}u \qquad (6)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r}\right) = -\frac{\partial p}{\partial r} + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2}\right\}v \qquad (7)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial z} = 0 \qquad (8)$$

An introduction of the following dimension less variables

$r' = r/a,\quad z' = z/a,\quad u' = u/c,\quad v' = \lambda v/ac,\quad t' = ct/\lambda,\quad p' = a^2p/\lambda c\mu,$
$h = h'/a = 1 + \phi\,\mathrm{Sin}\,2\pi z,\quad \phi = b/a.$

Into eqns. (6) – (8), yields after dropping primes

$$\delta\,\mathrm{Re}\left\{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r}\right\} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \delta^2\frac{\partial^2 u}{\partial z^2} \qquad (9)$$

$$\delta\,\mathrm{Re}\left\{\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r}\right\} = -\frac{\partial p}{\partial r} + \delta^2\left\{\frac{1}{r}\frac{\partial}{\partial r}(rv) - \frac{1}{r^2} + \delta^2\frac{\partial^2 v}{\partial z^2}\right\} \qquad (10)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial z} = 0 \qquad (11)$$

Where $R_e = \rho ca/\mu$ and $\delta = a/\lambda$ are respectively the Reynolds number and the wave number.
When the wavelength is large, the Reynolds number is quite small and therefore the inertial convective acceleration terms may be neglected (Shapiro et al., 1969). Using thus the long wavelength approximation (i.e., $\ll 1$), and neglecting the inertial term. Eqns. (9) – (11), take the form

$$\frac{dp}{dz} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u \qquad (12)$$

$$r = 0 \qquad (13)$$

The non-dimensional boundary conditions for the solution of the problem may now be stated (Beaver and Joseph, 1976, Srivastava et al., 2012) as

$$u = -1 + u_B \text{ at } r = h = 1 + \phi Sin 2\pi z \tag{14}$$

$$\frac{\partial u}{\partial r} = \frac{\alpha}{\sqrt{k}}(u_B - u_{porous}) \text{ at } r = h \tag{15}$$

$$\frac{\partial u}{\partial r} = 0 \text{ at } r = 0 \tag{16}$$

Where u_B is the non-dimensional slip velocity at the tube wall, α is the slip parameter, k is the Darcy number and $u_{porous} = -k\ dp/dz$.

ANALYSIS

In order to solve the problem analytically, we integrate equation (12) which yields

$$\frac{r}{2}\frac{dp}{dz} + C_1 = \frac{du}{dr} \tag{17}$$

The boundary condition on (16), i.e., $du/dr = 0$ at $= 0$, demands that

$$C_1 = 0$$

Substitution of $C_1 = 0$ into equation (17), derives

$$\frac{r}{2}\frac{dp}{dz} = \frac{du}{dr} \tag{18}$$

Integrating now equation (18), w.r.t. r, one obtains

$$\frac{r^2}{4}\frac{dp}{dz} + C_2 = u \tag{19}$$

An application of the boundary condition (14) into equation (19), yields

$$C_2 = (u_B - 1) - \frac{h^2}{4}\frac{dp}{dz} \tag{20}$$

Substituting C_2 into eqn. (19), we obtain the expression for velocity of the fluid as

$$u = -1 + u_B - \frac{1}{4}\frac{dp}{dz}\left(h^2 - r^2\right) \tag{21}$$

Now differentiating u with respect to r and using the boundary condition (15), we obtains

$$u_B = -k\frac{dp}{dz} + \frac{h\sqrt{k}}{2\alpha}\frac{dp}{dz} \tag{22}$$

Substitution of the value of u_B from eqn. (22) into the equation (21) yields the expression for velocity in to non-dimension form as:

$$u = -1 - \frac{1}{4}\frac{dp}{dz}[h^2 - r^2 + 4k - \frac{2h\sqrt{k}}{\alpha}] \tag{23}$$

The non-dimensional flow flux q is now derived as:

$$q = 2\int_0^h rudr \ = -h^2 - \frac{1}{8}\frac{dp}{dz}[h^4 + 8h^2k - 4h^3\sqrt{k}/\alpha] \tag{24}$$

Which yields:

$$-\frac{dp}{dz} = \frac{8(q+h^2)}{h^4 + 8h^2k - 4h^3\sqrt{k}/\alpha} \tag{25}$$

Following the report of Shapiro et al. (1969), the mean volume flow rate, Q at each cross-section over one period of the wave, is determined as

$$Q = q + 1 + \phi^2/2 \tag{26}$$

Substituting the value of q from (26) into the eqn. (25), one obtains
An use of relation (25) into equation (24), derives

$$-\frac{dp}{dz} = \frac{8(Q-1-\phi^2/2+h^2)}{h^4 + 8h^2k - 4h^3\sqrt{k}/\alpha} \tag{27}$$

The pressure drop, Δp = p (0) - p (1) across one wave length is calculated as:

$$\Delta p = \int_0^1 (-\frac{dp}{dz})dz$$
$$= 2\left[(Q-1-\phi^2/2)I_1 + I_2\right] \tag{28}$$

Where $$I_1 = 4\int_0^1 \frac{dz}{h^4 + 8h^2k - 4h^2\sqrt{k}/\alpha} \tag{29}$$

$$I_2 = 4\int_0^1 \frac{dz}{h^2 + 8k - 4h\sqrt{k}/\alpha} \tag{30}$$

The non-dimensional frication force, F is now obtained as:

$$F = \int_0^1 h^2(-\frac{dp}{dz})$$
$$= 2\left[(Q-1-\phi^2/2)I_2 + I_3\right] \tag{31}$$

Where $$I_3 = 4\int_0^1 \frac{dz}{1 + 8k/h^2 - 4\sqrt{k}/\alpha h} \tag{32}$$

The pressure-flow rate and the friction force-flow rate relationships from eqn. (28) and (31) are derived as:

$$Q = 1 + \phi^2/2 - \frac{I_2}{I_1} + \frac{\Delta p}{2I_1} \tag{33}$$

$$F = 2\left\{ I_3 - \frac{I_2{}^2}{I_1} + \frac{I_2}{I_1}\Delta p \right\} \tag{34}$$

The pressure rise $(-\Delta p)$ for zero time-mean flow and the time mean flow for zero pressure rise, which are of particular interest, are given as

$$(-\Delta p)_{Q=0} = 2\left[(1+\phi^2/2)I_1 - I_2 \right] \tag{35}$$

$$(Q)_{\Delta p=0} = 1 + \phi^2/2 - \frac{I_2}{I_1} \tag{36}$$

It is to note here that when the tube wall is impermeable (i.e., k=0), in the integrals involved in the results obtained in eqns. (28) and (31) become integrable in the derived form and one obtains the expressions for the pressure drop, Δp and the friction force, F as:

$$F = \frac{8}{(1-\phi^2)^{3/2}}\{Q - 1 - \phi^2/2 + (1-\phi^2)^{3/2}\} \tag{37}$$

$$F = \frac{8}{(1-\phi^2)^{3/2}}\{Q - 1 - \phi^2/2 + (1-\phi^2)^{3/2}\} \tag{38}$$

Which the same results are as obtained for the peristaltic flow of a Newtonian fluid in an axisymmetric tube by Shapiro et al. (1969).

NUMERICAL RESULTS AND DISCUSSION

To observe the quantitative effect of various parameters involved in the results of the study, computer codes are now developed to evaluate the analytical result obtained for the pressure drop, Δp and the friction force, F in equations (37) and (38), respectively. The parameters values are chosen as: Q (flow rate) = 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6; ϕ (amplitude ratio) = 0, 0.2, 0.4, 0.6; α (slip parameter) α = 0.1, 0.2, 0.3, 0.4, 0.5; and $K^{1/2}$ (here and after called Darcy number) = 0, 0.1, 0.2, 0.3, 0.4, 0.5. Some of the critical results obtained are displayed graphically in Fig. 2 - 13. It is worth mentioning that the present study corresponds to a two-dimensional channel case of Shapiro et al.(1969) and to classical Poiseceille flow for parameter values $K^{1/2}$ = 0 and ϕ = 0, $K^{1/2}$ = 0; respectively.

Fig. 2 reveals that the pressure drop, Δp decreases with increasing flow rate, Q for any value of the amplitude ratio, ϕ and for a given value of the slip parameter, α, and the Darcy number, $K^{1/2}$. One observe from Fig. 3 that for a given value of the slip parameter, α, the pressure drop, Δp, decreases with increasing flow rate, Q when channel walls are permeable(i.e.; $K^{1/2} \neq 0$). However, the flow characteristic Δp increase with flow rate, Q when channel walls are impermeable (i.e.; $K^{1/2}$ = 0) for any value of the amplitude ratio, ϕ. We notice from Fig. 4 that in the absence of the peristaltic waves, (i.e.; $\phi = 0$), the pressure drop, Δp decrease with increase flow rate, Q for small values of the slip parameter, α (in the absence of the peristaltic waves i.e.; $\phi = 0$) but increases with flow rate, Q for large values of the slip parameter, α. The flow characteristic, Δp decrease with Q for every value of the slip parameter, α in the presence of the peristaltic waves (i.e.; $\phi \neq 0$). From Fig. 5, one observe that when the channel walls are impermeable (i.e.; $K^{1/2} = 0$), the pressure drop, Δp decrease indefinite with amplitude ratio, ϕ, but increase with ϕ.

Fig. 6 reveals that the pressure drop, Δp decreases with the Darcy number, $K^{1/2}$ from its maximal magnitude at Darcy number $K^{1/2} = 0$ (impermeable channel walls) to a minimal value achieved in the range of $0.1 \leq K^{1/2} \leq 0$ and afterwards it approaches to an asymptoric magnitude with increasing Darcy number, $K^{1/2}$. For a given value of the Darcy number, α, the pressure drop Δp decreases assumes a minimal value and the increases in the range of values of the slip parameters $0.1 \leq \alpha \leq 0.2$. The flow characteristic, Δp increases steeply beyond this range and achieved a maximal and afterwards again deceases rapidly with increasing slip parameter, α foe any value of the amplitude ratio, ϕ and the flow rate Q,(Fig. 7).

One observe that the friction force, F decreases with increasing flow rate Q for a given value of the slip parameter, α and the Darcy number, $K^{1/2}$(Fig. 8) one notices from Fig.9 that friction force, F decreases with increasing flow rate, Q in the case of permeable channel walls (i.e.; $K^{1/2} \neq 0$) but increase when channel walls are impermeable (i.e.; $K^{1/2} = 0$). From Fig. 10, we observe that the friction force, F decreases with increasing flow rate, Q but increases with the flow rate, Q in the absence of the peristaltic waves (i.e.; $\phi = 0$) .However. The flow characteristic, F decreases with increasing flow rate, Q in the presence of the peristaltic waves (i.e.; $\phi \neq 0$). The friction force, F decreases indefinitely with increasing amplitude ratio, ϕ for impermeable channel walls (i.e.; $K^{1/2} = 0$), but the flow characteristic, F increases with amplitude ratio, ϕ, when the channel walls are permeable (i.e.; $K^{1/2} \neq 0$, Fig. 11). An inspection of Fig. 12 reveals that the friction force, F deceases from its maximal values at Darcy number, $K^{1/2} = 0$ to a minimal magnitude achieved in the range of values of slip parameter $0.1 \leq \alpha \leq 0.2$ and afterwards approaches to an asymptotic value with increasing slip parameter, α. The friction force, F declasses the flow characteristic, ΔF increases steaply beyond this range and achieved a maximal and afterwards again decreases rapidly with increasing slip parameter, α for any given values of the amplitude ratio, ϕ, and the flow rate, Q,(Fig. 13).

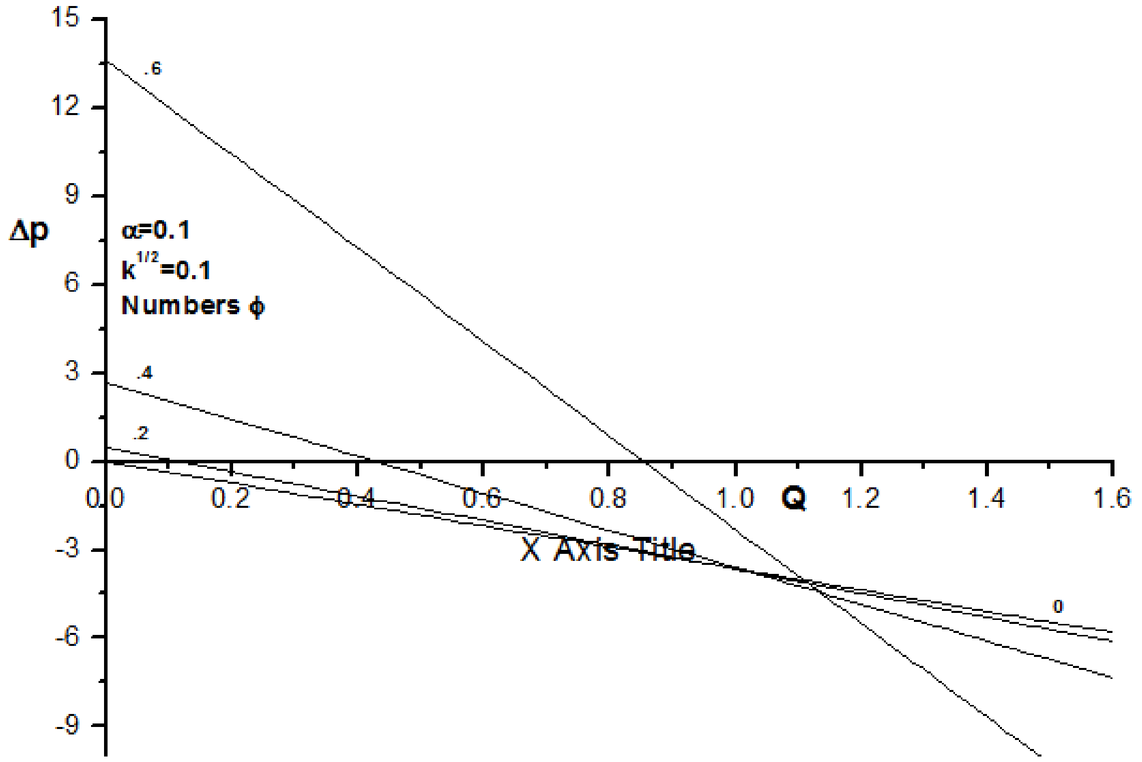

Fig. 2 Variation of pressure drop, Δp with flow rate, Q for different $\emptyset$.

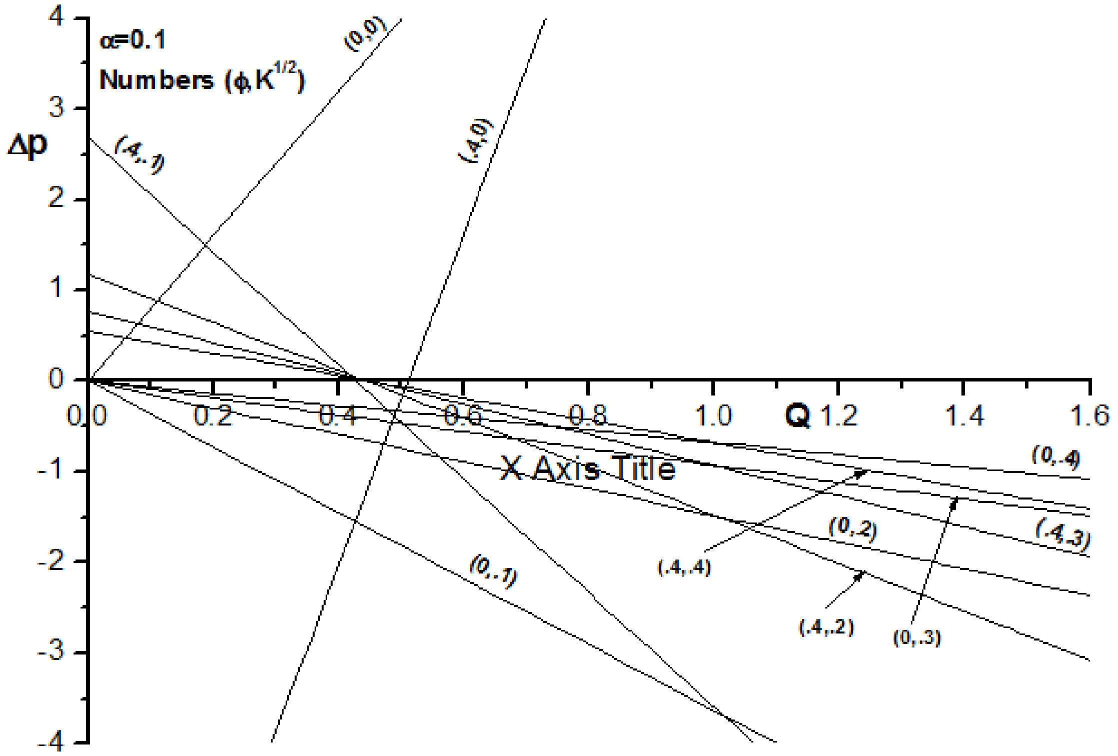

Fig. 3 Variation of pressure drop, Δp with flow rate, Q for different Darcy number $k^{1/2}$.

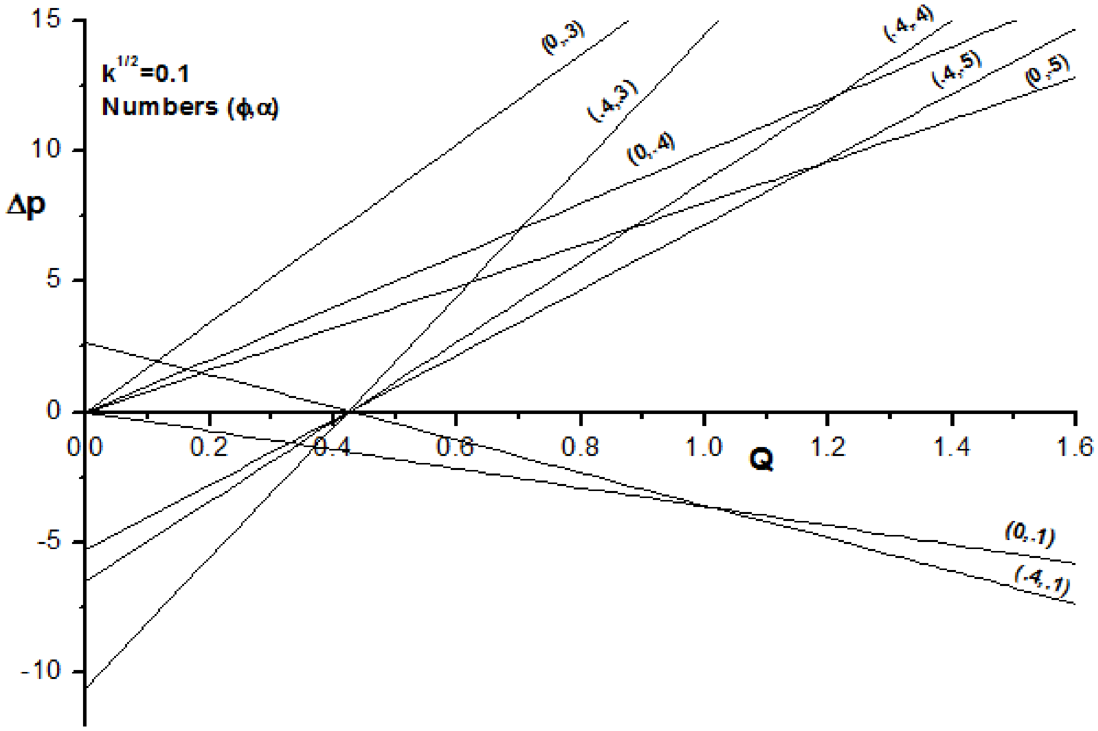

Fig. 4 Variation of pressure drop, Δp with flow rate, Q for different slip parameter, α.

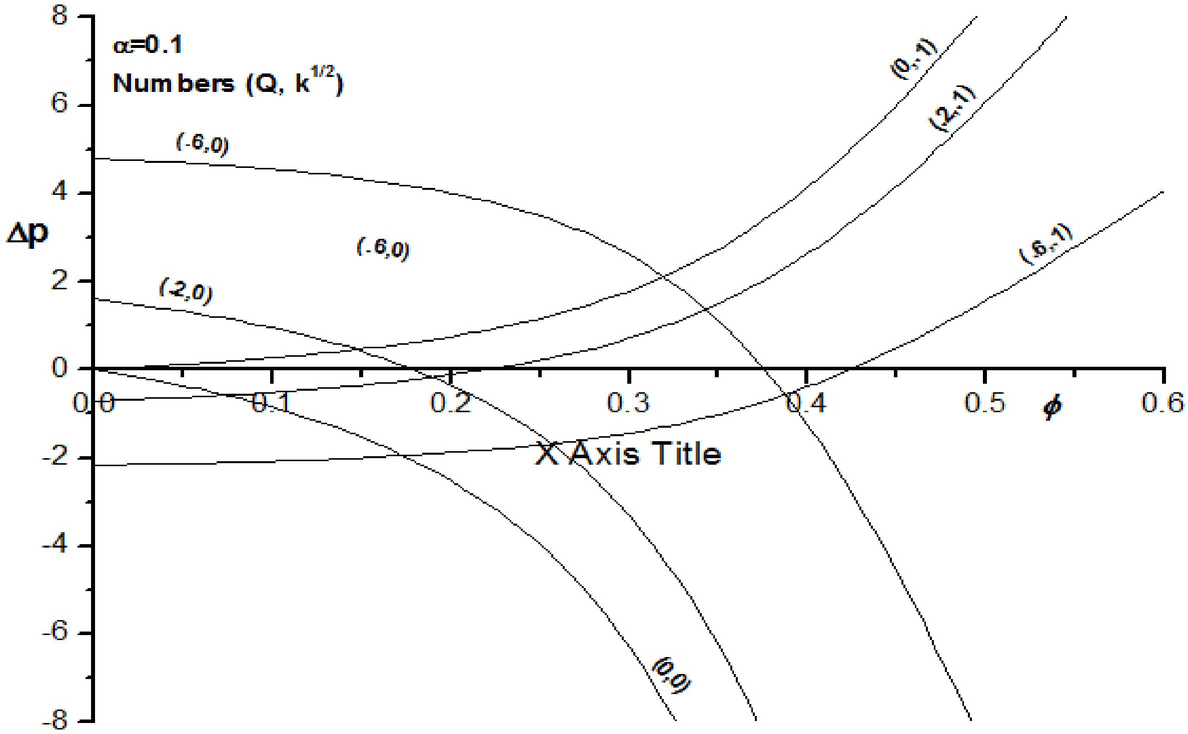

Fig. 5 Variation of pressure drop, Δp with amplitude ratio, $\emptyset$ for different Q and $k^{1/2}$.

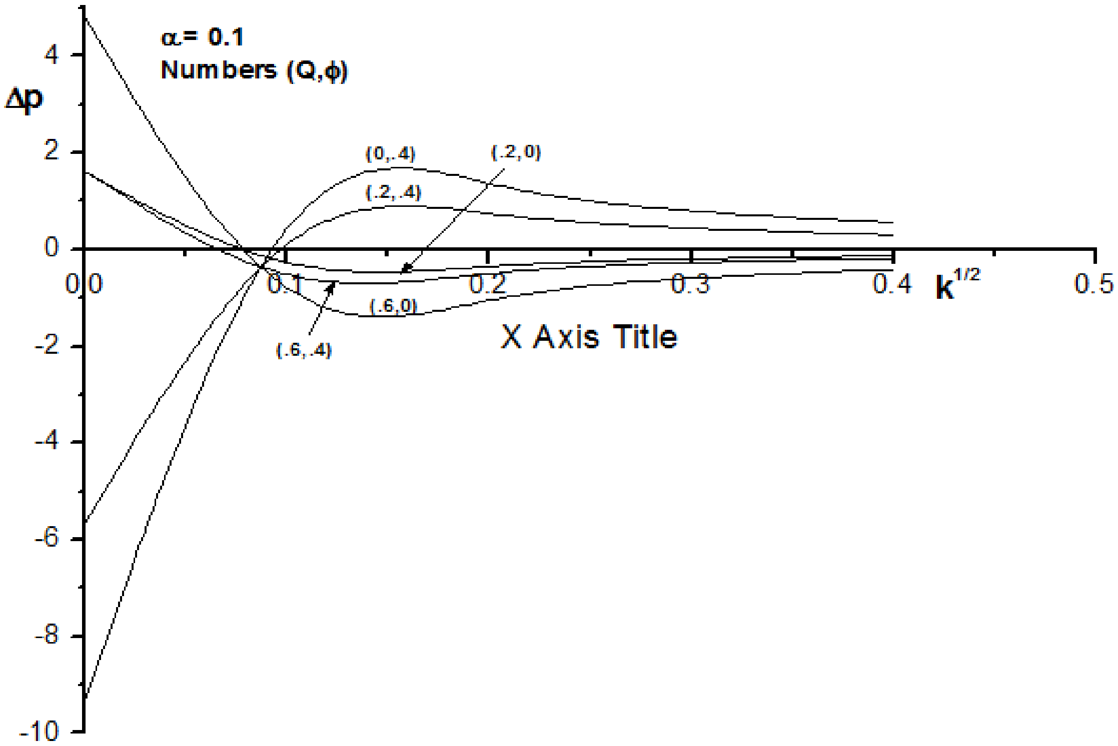

Fig. 6 Variation of pressure drop, Δp with Darcy number, $k^{1/2}$ for different Q and $\emptyset$.

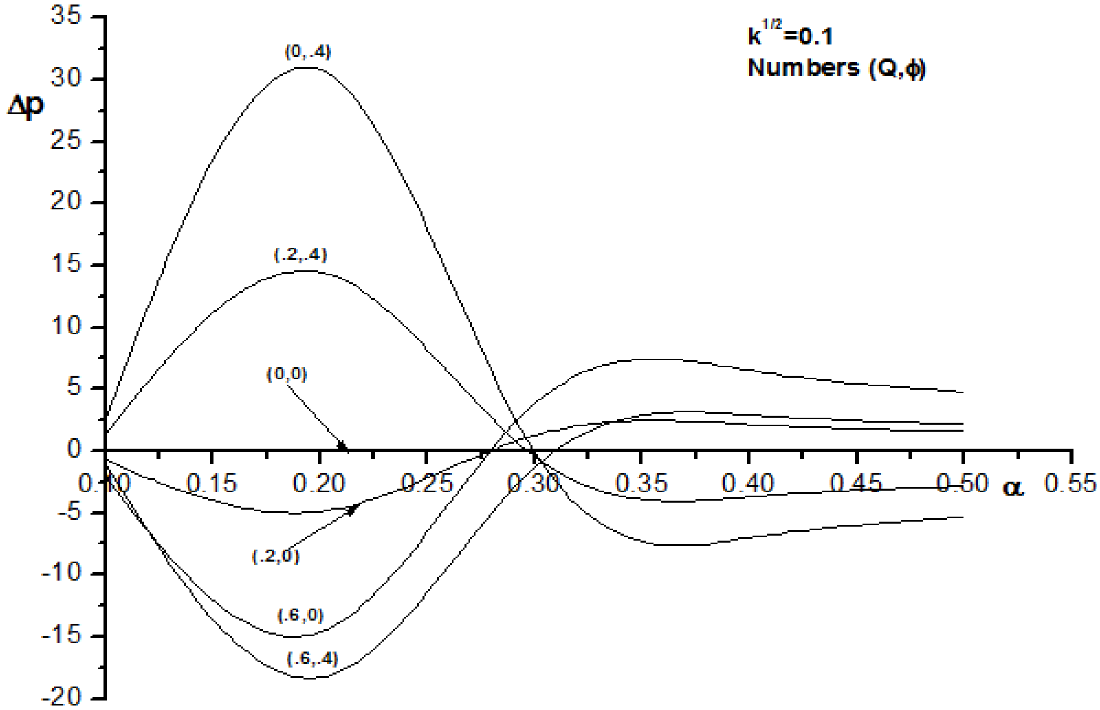

Fig. 7 Variation of pressure drop, Δp with slip parameter, α for different Q and $\emptyset$.

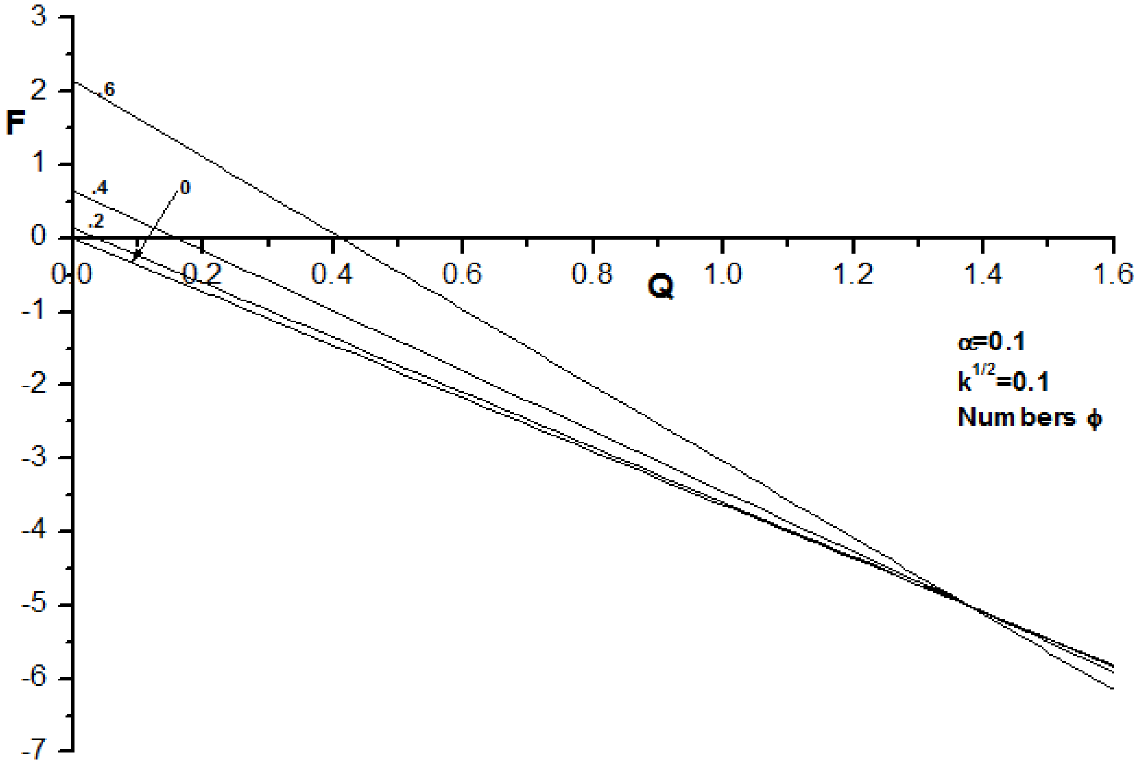

Fig. 8 Variation of friction force, F with flow rate, Q for different ∅.

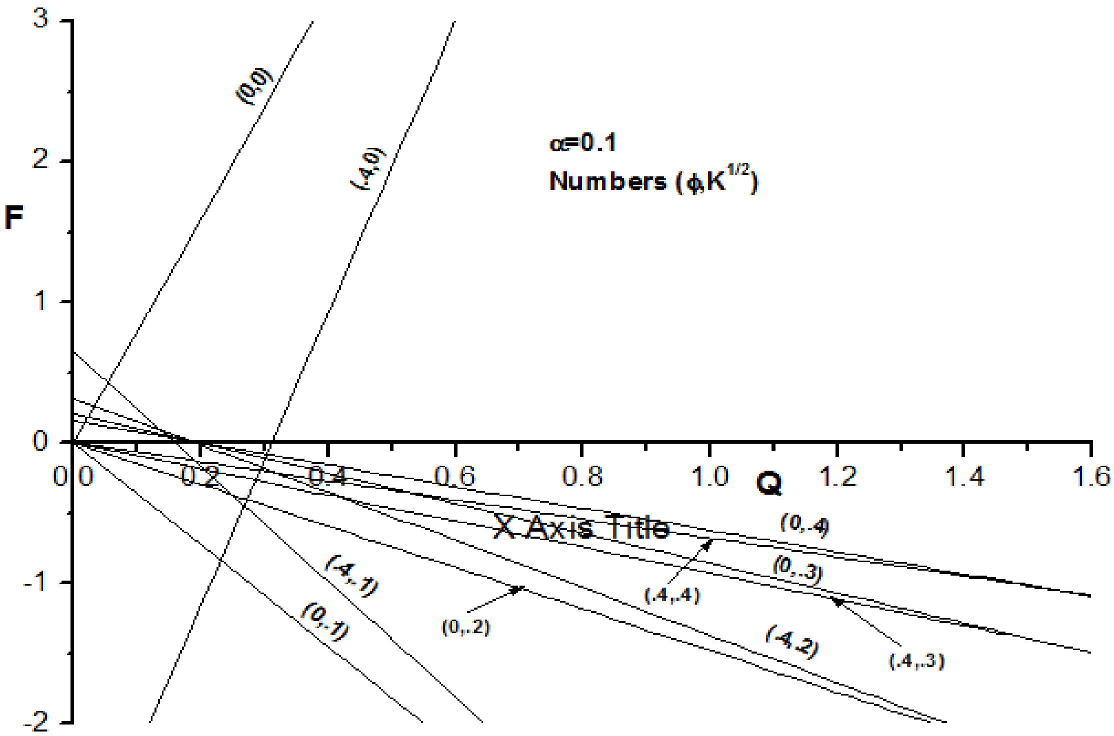

Fig. 9 Variation of friction force, F with flow rate, Q for different Darcy number, $k^{1/2}$.

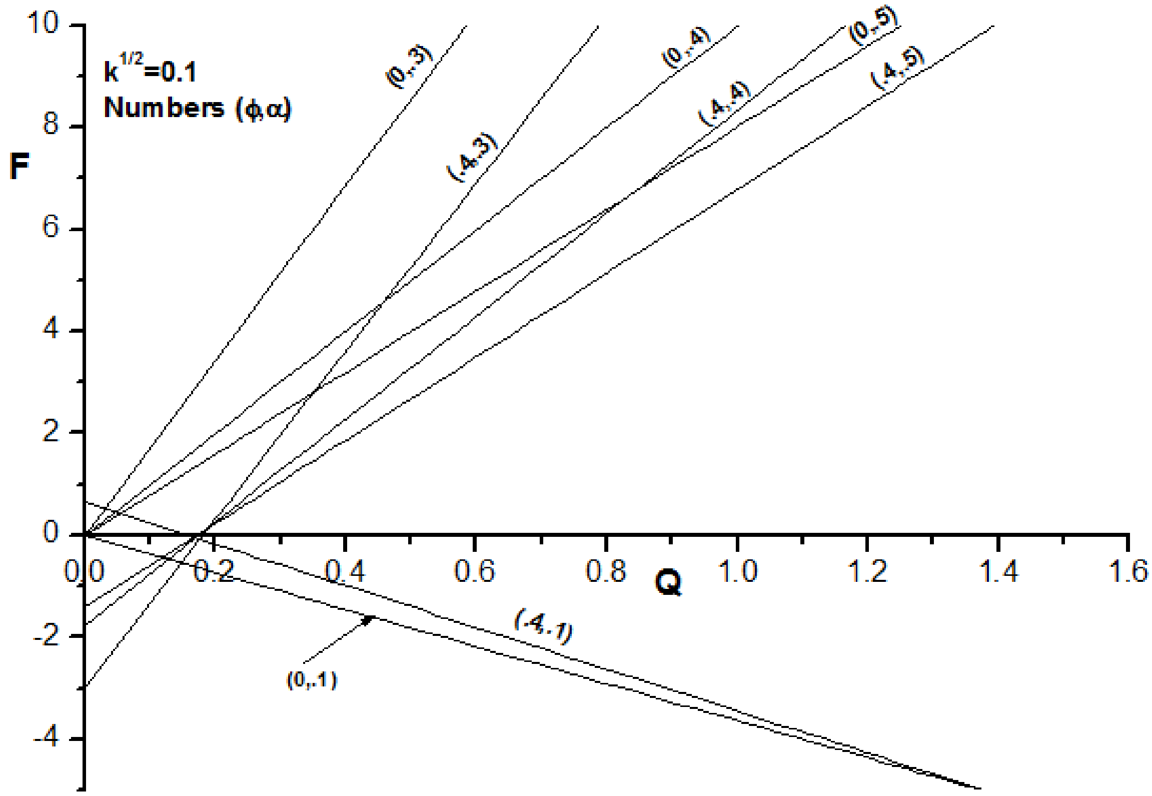

Fig. 10 Variation of friction force, F with flow rate, Q for different slip parameter,α.

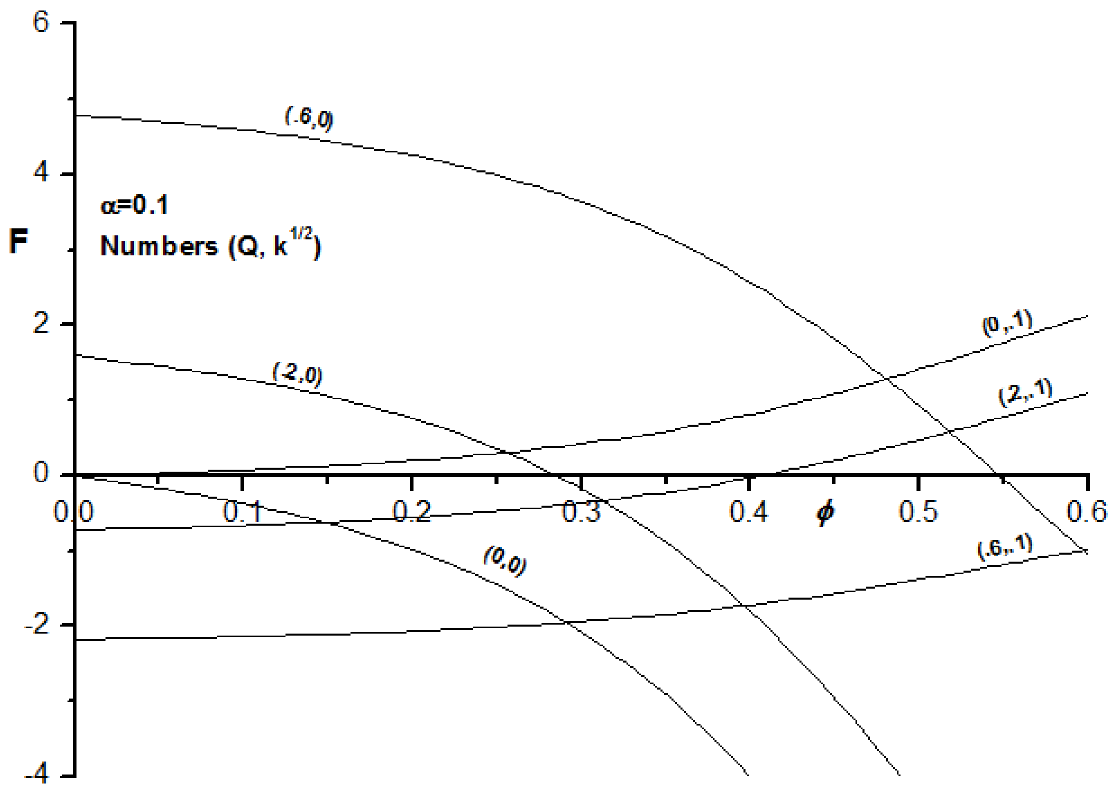

Fig. 11 Variation of friction force, F with amplitude ratio, Ø for different Q and $k^{1/2}$.

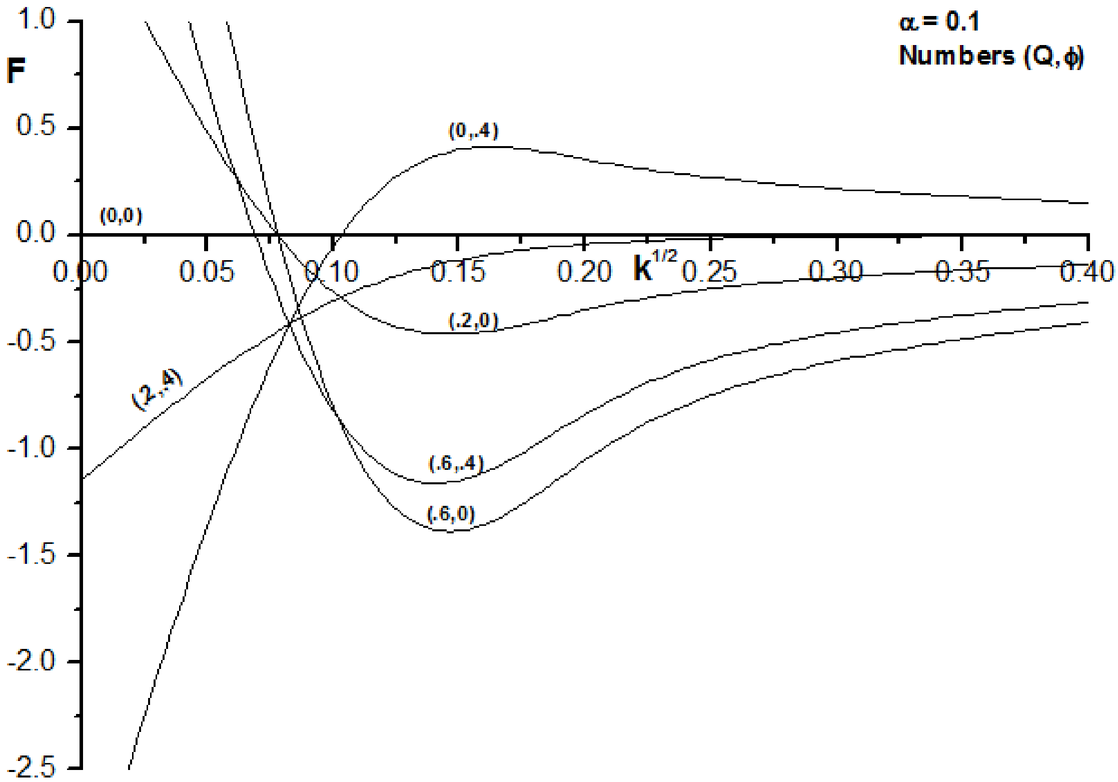

Fig. 12 Variation of friction force, F with Darcy number $k^{1/2}$ for different Q and ∅.

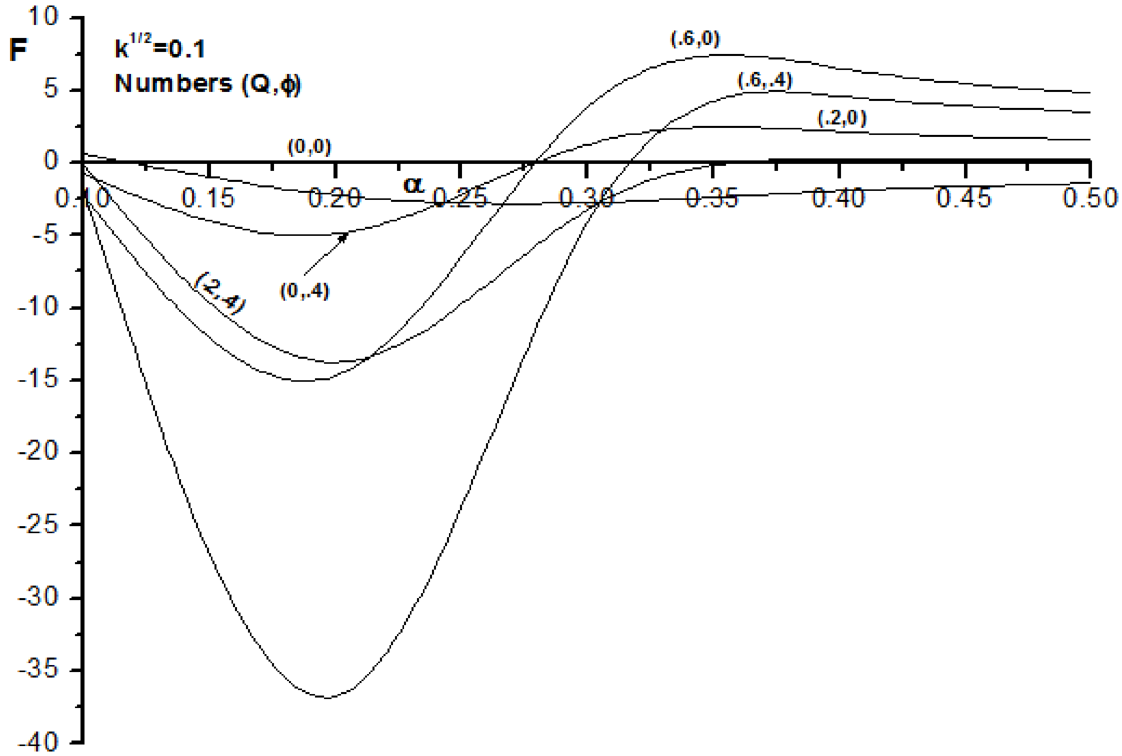

Fig. 13 Variation of friction force, F with slip parameter, α for different Q and ∅.

CONCLUSIONS

Quantitative effect of parameters such as pressure drop and the friction force have been obtained for certain values of flow rate, amplitude ratio, slip parameter and Darcy number.

The graphical results reveal that the pressure drop decreases with increasing flow rate for any value of the amplitude ratio and for a given value of the slip parameter and Darcy number. It is further observed that for a given value of slip parameter, pressure drop decreases with increasing flow rate when the tube wall are permeable but the pressure drop increase with flow rate when tube walls are impermeable for any value of the amplitude ratio. Further observations revel that when the tube walls are impermeable the pressure drop decreases indefinitely with amplitude ratio but it increases with amplitude ratio. During the above study variation of pressure drop with flow rate in the absence and presence of peristaltic waves have been taken up. We notice that in the absence of the peristaltic waves the pressure drop decreases with increase in flow rate for small values of the slip parameter but increases with flow rate for large values of the slip parameter. However the flow characteristic decreases for every value of the slip parameter in the presence of the peristaltic waves.

The plot depicting variation of pressure drop with Darcy number reveals that the pressure drop decreases with the Darcy number from its maximum magnitude for impermeable tube walls to a minimum value and thereafter it approaches to an asymptotic magnitude with increasing values of Darcy number. For a given value of the Darcy number the pressure drop decreases assumes a minimal value and then increases in a specific range of values of the slip parameters. The flow characteristic increases steeply beyond this range which attains a maximum value and again steeply decreases with increasing slip parameter for any value of the amplitude ratio and flow rate.

Friction force decreases with increasing flow rate for a given value of the slip parameter and Darcy number it also decreases with increasing flow rate in the case of permeable tube walls but increases when tube walls are impermeable. From the graphical variations it is observed that the friction force decreases with increasing flow rate but increases with the flow rate in the absence of the peristaltic waves. The flow characteristic (Friction Force) decreases with increasing flow rate in the presence of the peristaltic waves. The friction force decreases indefinitely with increasing values of amplitude ratio for impermeable tube walls but it increases with amplitude ratio when the tube walls are permeable. It is further observed that the friction force deceases from its maximum value to a minimal magnitude for a specific range of values of slip parameter and thereafter approaches to an asymptotic value with increasing slip parameter. The friction force increases steeply beyond this range and achieves a maximum value after which it again decreases rapidly with increasing slip parameter values for any given values of the flow rate and amplitude ratio.

REFERENCES

1. El Shehawey EF, Mekheimer Kh S, Kaldas S F, Afifi N A S. Peristaltic transport through a porous medium. J Biomath.1999; 14

2. Caro, C. G. Pedley, T. J., Schroter, R.C. and Seed, W. A. (1978). The Mechanics of the Circulation. Oxford Medical, N. Y.

3. G. Radhakrishnamacharya, Long wavelength approximation to peristaltic motion of a power law fluid, Rheologica Acta, vol.21, pp.30-35, 1982.

4. Shapiro, A. H., Jaffrin, M. Y. and Weinberg, S. L. Peristaltic pumping with long wavelength at low Reynolds number. J. Fluid Mech. 37, 799-825, 1969**.**

5. El Shehawey EF, Sobh AMF, Elbarbary EME. Peristaltic motion of a generalized Newtonian fluid through a porou medium. Phys Soc Jpn. 2000; 69: 401-407.

6. Jaffrin, M.Y. and Shapiro, A.H. Peristaltic Pumping, Ann. Rev. Fluid Mech., 3(1971), 13-36.

7. Latham, T.W. (1966). Fluid Motion in a Peristaltic Pump. M.S. Thesis, M.I.T. Mass.

8. Rao, A. R. and Usha, S. (1995). Peristaltic transport of two immicible fluids with long
wavelength at low Reynolds number, J. Fluid Mech., 298, pp. 271-285.

9. Merriam-Webster's (1995), Medical Dictionary, Merriam-Webster's, Inc.

10. Srivastava, L.M. and Srivastava, V.P. (1984). Peristaltic transport of blood: Casson model II. J.Biomech, Vol. 17, pp. 821-829.

11. Beavers, G.S. and Joseph, D. D. (1967). Boundary conditions at a naturally permeable wall. J.Fluid Mech. 30(1), pp. 197-207.

12. Srivastava, L.M. and Srivastava, V.P. Peristaltic transport of power law fluid: application to the ductus of efferuntus of the reproductive tract, Rheol. Act a., 27(1988), 428-433.

13. Misra, J.C. and Pandey, S.K., "Peristaltic flow of a multi layered power-law fluid through a cylindrical tube", Int. J. Engng. Sci. 39 (2001), 387-402.

14. Siddiqui, A.M., Provost, A. and Schwarz, W.H. Peristaltic pumping of a second-order fluid in a planar channel, Rheol.Acta, 30(1991), 249-262.

15. Shapiro, A.H., Jaffrin, M.Y and Weinberg, S.L. Peristaltic pumping with long wavelengths at low Reynolds number, J. Fluid Mech. 37(1969), 799-825.
16. Shukla, J.B. and Gupta, S.P. Peristaltic transport of a power-law fluid with variable
Consistency, Trans. ASME J.Biomech.Engng., 104(1982), 182-186.
17. Srivastava, V.P. (2007b). A Theoretical model for Blood Flow in Small Vessels, Applications and Applied Mathematics, Vol. 2, pp. 51-65.