TWO-PHASE ARTERIAL BLOOD FLOW THROUGH A COMPOSITE STENOSIS

Amit Medhavi, Rupesh K. Srivastav[1], Qazi Shoeb Ahmad[1] and V. P. Srivastava[2]

Department of Mech. Engng, Kamla Nehru Institute of Technology Sultanpur-228118, India.
[1]Department of Mathematics, Integral University, Lucknow, India.
[2]Director, Krishna Girls Engineering College, Kanpur-209217, India.

Abstract- The effects of a composite stenosis on blood flow characteristics in an artery have been investigated. The flowing blood has been represented by a macroscopic two-phase model (i.e., a suspension of red cells in plasma). The expressions for the flow characteristics, namely, the impedance, the wall shear stress and the shear stress at the stenosis throat have been obtained. The effects of hematocrit on these flow characteristics have been discussed thoroughly and briefly.

Key Words: Hematorit, impedance, shear stress, stenosis throat, erythrocytes.

INTRODUCTION

Stenosis or arteriosclerosis is an abnormal and unnatural growth that develops at various locations of the cardiovascular system under diseased conditions, which occasionally results into serious consequences (Srivastava, 1995). The actual cause for the development of the frequently occurring cardiovascular disease, stenosis is related to the nature of blood movement and the mechanical behavior of the blood vessel walls. It is well known that the fluid dynamical parameters, particularly the high wall shear stress play an important role in the genesis of the disease, although the root causes of the formation of stenotic lesions are not well understood. It is well established that once the constriction has developed, it brings about the significant alterations in the flow field, pressure distribution, wall shear stress and the flow resistance (impedance). With the advent of the discovery that haemodynamic factors play an important role in the genesis and proliferation of the disease has attracted the early investigators including Young (1968), Young and Tsai (1973), Deshpande et.al. (1976), Caro et.al. (1978), Ahmed and Giddens (1983), and several others to study the blood flow through local constrictions, since the first investigation of Mann et.al. (1938). A brief account of researches on the topic, reported so far, may be had from Young (1979), Srivastava (1995), Sarkar and Jayaraman (1998), Ku (1997), Ponalagusamy (2007), Mishra and Verma (2007), Mekheimer and El-Kot (2008), etc. The recent years investigation includes the studies of Sankar and Lee (2009), Srivastava and coworkers (2009, 2010), Singh et al. (2010), Biswas and Chakraborty (2010a,b), Medhavi (2011), Mishra and Siddiqui (2011), Nadeem et al. (2011), Mekheirmer et. al. (2011), Ponalagusamy and Selvi (2011), Bandyopadhyay and Layek (2011, 2012), Srivastava et al. (2012) and many others.

It is known that blood behaves like a non-Newtonian fluid (Merill et al., 1965; Charm and Kurland, 1965, 1974; Hershey, et al., 1964; Huckaba and Hahu, 1968) at low shear rates. The theoretical study of Haynes (1960) and experimental observations of Cokelet (1972), however, indicate that blood can no longer be treated as a single-phase homogeneous viscous fluid in narrow arteries (of diameter $\leq 1000\ \mu m$). The individuality of red cells (of diameter $8\ \mu m$) is important in such large vessels with diameter up to 100 cells diameter (Srivastava and Srivastava, 1983). A brief survey of the literature on multiphase blood flow has been presented by

Srivastava (2007). A survey of the literature on arteriosclerotic development reveals that the studies conducted are mainly concerned with the single symmetric and non-symmetric stenoses. However, the stenosis may develop in series (multiple stenoses) or may be of irregular shapes or bell shaped or of composite in nature, etc. An attempt is made in the present investigation to explore the effects of a composite stenosis on the flow characteristics of blood taking into account that the flowing blood is represented by a macroscopic two-phase fluid (i.e., a suspension of erythrocytes in plasma).

FORMULATION OF THE PROBLEM

Consider the axisymmetric flow of blood through a composite stenosis in an artery of circular cross-section. The geometry of the composite stenosis, assumed to be manifested in the arterial wall segment is described (Fig.1) as

$$R(z)/R_0 = 1 - \frac{2\delta}{R_0 L_0}(z-d); \qquad d \le z \le d + L_0/2, \tag{1}$$

$$= 1 - \frac{\delta}{2R_0}\left\{1 + \cos\frac{2\pi}{L_0}(z - d - L_0/2)\right\}; \; d + L_0/2 \le z \le d + L_0, \tag{2}$$

$$= 1; \qquad \text{otherwise}, \tag{3}$$

where $R \cong R(z)$ and R_0 are respectively, the radius of the artery with and without stenosis, L_0 is the length of the stenosis and d indicates its location, δ is the maximum projection in the lumen located at $z=d+L_0/2$.

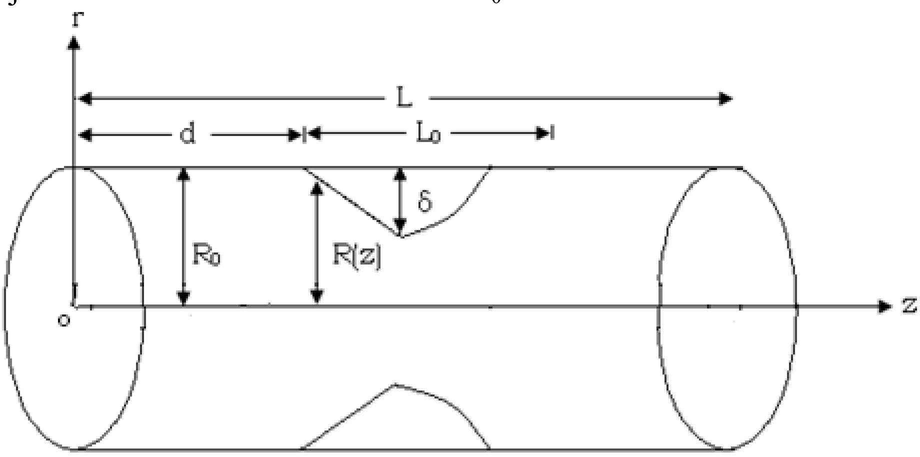

Fig. 1 Geometry of a composite stenosis in an artery.

Blood is assumed to be represented by a two-phase macroscopic model, that is, a mixture of plasma and erythrocytes (red cells). The equations describing the steady flow of a two-phase macroscopic model of blood may be expressed (Srivastava and Srivastava, 1983, 1989) as

$$(1\text{-}C)\,\rho_f\left\{u_f\frac{\partial u_f}{\partial z} + v_f\frac{\partial u_f}{\partial r}\right\} = -\,(1\text{-}\,C)$$

$$\frac{\partial p}{\partial z} + (1 - C)\,\mu_s\,(C)\,\nabla^2 u_f$$

$$+CS(u_p - u_f), \quad (2)$$

$$(1\text{-}C)\ \rho_f \left\{ u_f \frac{\partial v_f}{\partial z} + v_f \frac{\partial v_f}{\partial r} \right\} = -(1\text{-}C)$$

$$\frac{\partial p}{\partial r} + (1-C)\ \mu_s(C)\ (\nabla^2 - \frac{1}{r^2})\, v_f$$

$$+CS\,(v_p - v_f), \quad (3)$$

$$\frac{\partial}{\partial r}[(1-C)v_f] + (1-C)\frac{v_f}{r} + \frac{\partial}{\partial z}[(1-C)u_f] = 0, \quad (4)$$

$$\rho_p \left\{ u_p \frac{\partial u_p}{\partial z} + v_p \frac{\partial u_p}{\partial r} \right\} = -C\frac{\partial p}{\partial z} + CS\,(u_f - u_p), \quad (5)$$

$$\rho_p \left\{ u_p \frac{\partial v_p}{\partial z} + v_p \frac{\partial v_p}{\partial r} \right\} = -C\frac{\partial p}{\partial r} + CS\,(v_f - v_p), \quad (6)$$

$$\frac{\partial}{\partial r}[C\,v_p] + \frac{Cv_p}{r} + \frac{\partial[C\,u_p]}{\partial z} = 0, \quad (7)$$

where $\nabla^2 = \partial/\partial r^2 + (1/r)(\partial/\partial r) + \partial^2/\partial z^2$ as a two-dimensional Laplacian operator, r is the radial coordinate measured perpendicular to the axis of the tube. (u_f, v_f) and (u_p, v_p) are the (axial, radial) components of the fluid and particle velocities, respectively, C denotes the volume fraction density of the particles, p is the pressure, $\mu_s(C) \simeq \mu_s$ is the mixture viscosity (apparent or effective viscosity) , S is the drag coefficient of interaction for the force exerted by one phase on the other, ρ_f and ρ_p are the actual density of the material constituting the fluid (plasma) and the particle (erythrocyte) phases, respectively, (1-C)ρ_f is the fluid phase and $C\rho_p$ is particle phase densities, and the subscripts f and p denote the quantities associated with the plasma (fluid) and erythrocyte (particle) phases, respectively. The expressions for drag coefficient of interaction, S and the viscosity of the suspension, μ_s for the present study are selected (Srivastava and Srivastava, 2009; Charm and Kurland, 1974) as

$$S = \frac{9}{2}\frac{\mu_o}{a_o^2}\ \frac{4 + 3[8C - 3C^2]^{1/2} + 3C}{(2-3C)^2}, \quad (8)$$

$$\mu_s\ (C) = \frac{\mu_o}{1 - mC},$$

$$m = 0.070 \exp[2.49C + (1107/T)\exp(-1.69C)], \quad (9)$$

where T is the measure in absolute scale of temperature (^oK) , μ_o is the constant plasma viscosity and a_o is the radius of a red cell.

The equations governing the laminar, steady, one-dimensional flow of blood in an artery in the case of a mild stenosis (i.e., $\delta/R_0 << 1$) are derived (Young ,1968 ; Srivastava and Rastogi, 2009), from equations (2)-(7) as

$$(1\text{-C})\ \frac{dp}{dz} = (1-C)\frac{\mu_s}{r}\ \frac{\partial}{\partial r}\ (r\frac{\partial}{\partial r})\ u_f + CS\ (u_p - u_f), \tag{10}$$

$$C\ \frac{dp}{dz} = CS\ (u_f - u_p)\ , \tag{11}$$

The boundary conditions for the problem are now stated as

$$\frac{\partial u_f}{\partial r} = 0 \text{ at } r = 0, \tag{12}$$

$$u_f = 0 \quad \text{ at } r = R(z), \tag{13}$$

ANALYSIS

An integration of equations (10) and (11) under the boundary conditions (12) and (13), yields the expressions for velocity profiles, u_f and u_p as

$$u_f = -\frac{R_0^2}{4(1-C)\,\mu_s}\ \frac{dp}{dz}\left\{(R/R_0)^2 - (r/R_0)^2\right\}\ , \tag{14}$$

$$u_p = -\frac{R_0^2}{4(1-C)\,\mu_s}\ \frac{dp}{dz}\left\{(R/R_0)^2 - (r/R_0)^2 + \frac{4(1-C)\mu_s}{SR_0^2}\right\}. \tag{15}$$

The volumetric flow rate, Q is now calculated as

$$Q = 2\pi\ (1\text{-}C)\int_0^R r\ u_f\ dr + 2\pi\ C\int_0^R r\ u_p\ dr$$

$$= -\frac{\pi R_0^4}{8(1-C)\,\mu_s}\ \frac{dp}{dz}\left[(R/R_0)^4 + \beta(R/R_0)^2\right] \tag{16}$$

or

$$\frac{dp}{dz} = -\frac{8(1-C)\,\mu_s\ Q}{\pi\ R_0^4}\ \varphi(z), \tag{17}$$

with $\beta = 8C(1-C)\mu_o/SR_0^2$, a non-dimensional suspension parameter, and $\varphi(z) = 1/F(z)$, $F(z) = (R/R_0)^4 + \beta(R/R_0)^2$.

The pressure drop, $\Delta p\ (= p \text{ at } z = \text{-}L, - p \text{ at } z = L)$ across the stenosis in the tube of length, L is obtained as

$$\Delta p = \int_{-L}^{L}\left(-\frac{dp}{dz}\right)dz = \frac{8(1-C)\,\mu_s\ Q}{\pi\ R_0^4}\,\psi\ , \tag{18}$$

where

$$\psi = \int_0^d [\varphi(z)]_{R/R_0=1} dz + \int_d^{d+L_o/2} [\varphi(z)]_{R/R\,\mathrm{from}\,(1)} dz + \int_{d+L_o/2}^{L_o} [\varphi(z)]_{R/R_0\,\mathrm{from}(2)} dz + \int_{L_o}^{L} [\varphi(z)]_{R/R_0=1} dz , \tag{19}$$

The closed form analytical evaluation of the second and the third integrals in the expression for ψ obtained above are almost formidable tasks and therefore shall be evaluated numerically. However, the analytical evaluation of the first and the fourth integrals are straight forward. Using the definitions from the published literature (Young, 1968; Srivastava and Rastogi, 2009), the expressions for the impedance (flow resistance), λ, the wall shear stress in the stenotic region, τ_w, and the shearing stress at the stenosis throat, τ_s are derived as

$$\lambda = (1\text{-}C)\mu\left[\frac{1\text{-}L_0/L}{1+\beta} - \frac{R_0L_0}{2\beta\delta L}\left\{1 - \frac{1}{1-\delta/R_0} + \frac{1}{\beta}\tan^{-1}\frac{\delta\sqrt{\beta}/R_0}{1+\beta-\delta/R_0}\right\} + \frac{L_0}{2\pi L}\int_0^\pi \frac{d\theta}{(a+b\cos\theta)^2\left[(a+b\cos\theta)^2+\beta\right]}\right], \tag{20}$$

$$\tau_w = \frac{(1-C)}{(R/R_0)^3 + \beta(R/R_0)}, \tag{21}$$

$$\tau_s = \frac{(1-C)\mu}{(1-\delta/R_0)^3 + \beta(1-\delta/R_0)}, \tag{22}$$

where
$$\lambda = \frac{\bar{\lambda}}{\lambda_0},\ (\tau_w, \tau_s) = \frac{(\bar{\tau}_w, \bar{\tau}_s)}{\tau_0},$$
$$\bar{\lambda} = \frac{\Delta p}{Q},\ \bar{\tau}_w = -\frac{R}{2}\left(\frac{dp}{dz}\right),$$
$$\bar{\tau}_s = \left[-\frac{R}{2}\left(\frac{dp}{dz}\right)\right]_{R/R_0 = (1-\delta/R_0)},$$
$$\mu = \frac{\mu_s}{\mu_0},\ \lambda_0 = \frac{8\mu_0 L}{\pi R_0^4},\ \tau_0 = \frac{4\mu_0 Q}{\pi R_0^3},\ a = 1\text{-}\frac{\delta}{2R_0},\ b = \frac{\delta}{2R_0},$$
$$\theta = \pi\text{-}\frac{2\pi}{L_0}(z - d - \frac{L_0}{2}).$$

λ_0 and τ_0 are the flow resistance and wall shear stress for a normal artery (no stenosis) in the absence of the particle phase (i.e. C = 0, Newtonian fluid).

In the absence of the particles (i.e. C=0), the results for a Newtonian fluid are derived from equation (20)-(22), as

$$\lambda_N = 1 - \frac{L_0}{L} - \frac{R_0L_0}{6\delta}\left[1 - \frac{1}{(1-\delta/R_0)^3}\right] + \frac{L_0}{2\pi L}\int_0^\pi \frac{d\theta}{(a+b\cos\theta)^4}, \tag{23}$$

$$\tau_{wN} = \frac{1}{(R/R_0)^3}, \tag{24}$$

$$\tau_{sN} = \frac{1}{(1 - \delta/R_0)^3}, \tag{25}$$

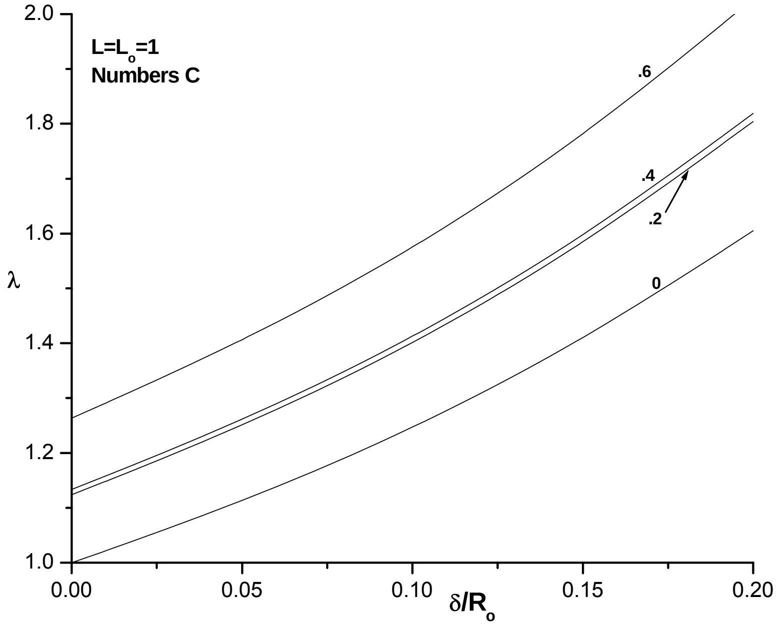

Fig. 2 Impedance, λ versus stenosis height, δ/R_o for different hematocrit, C.

NUMERICAL RESULTS AND DISCUSSIONS

Computer codes are now developed to evaluate analytical results obtained in equations (20)-(22) at the temperature of 37°C in a tube of radius 0.01cm in order to observe the quantitatively effects of the hematocrit and other parameters on the blood flow characteristics for various parameter values (Young,1968; Srivastava, 1995 and Srivastava and Rastogi, 2009): d(cm) = 0; L_0 (cm) = 1; L(cm) = 1, 2, 5 ; C = 0, 0.2, 0.4, 0.6; δ/R_0=0, 0.05, 0.10, 0.15, 0.20. It is worth mentioning that present study corresponds to the case of a Newtonian fluid and no stenosis for parameter values C=0 and $\delta/R_0 = 0$, respectively.

For any given stenosis height, δ/R_0, the impedance, λ increases with the hematocrit, C, and also for any given hematocrit, C, λ increases with the stenosis height, δ/R_0 (Fig.2). The blood flow characteristic, λ decreases with the increasing tube length, L for any given stenosis height, δ/R_0 and hematocrit, C. This in turn implies that the impedance, λ increases with the stenosis length, L_o for any given set of other parameters (Fig.3). One notices that the blood flow characteristic, λ increases steeply with the hematocrit, C for any given stenosis height, δ/R_0 (Fig.4).

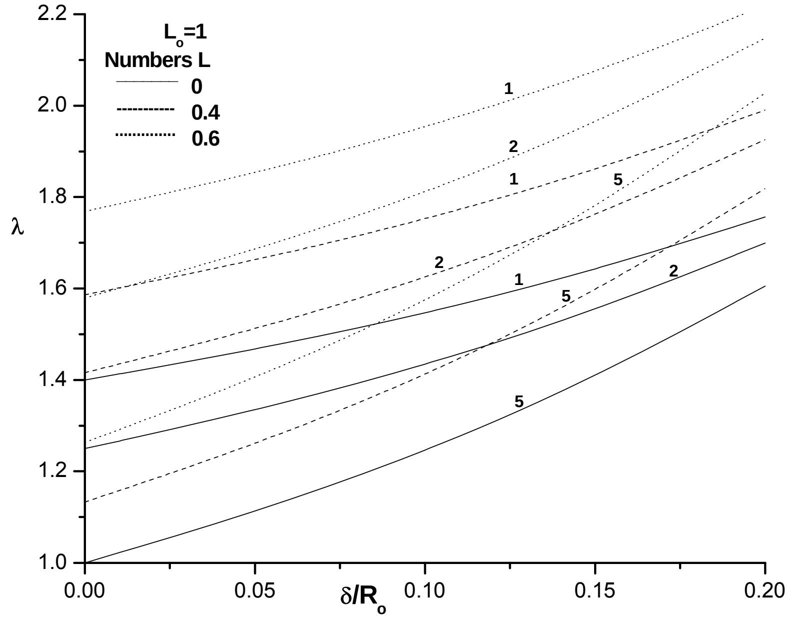

Fig. 3 Impedance, λ versus stenosis height, δ/R_o for different artetry length, L.

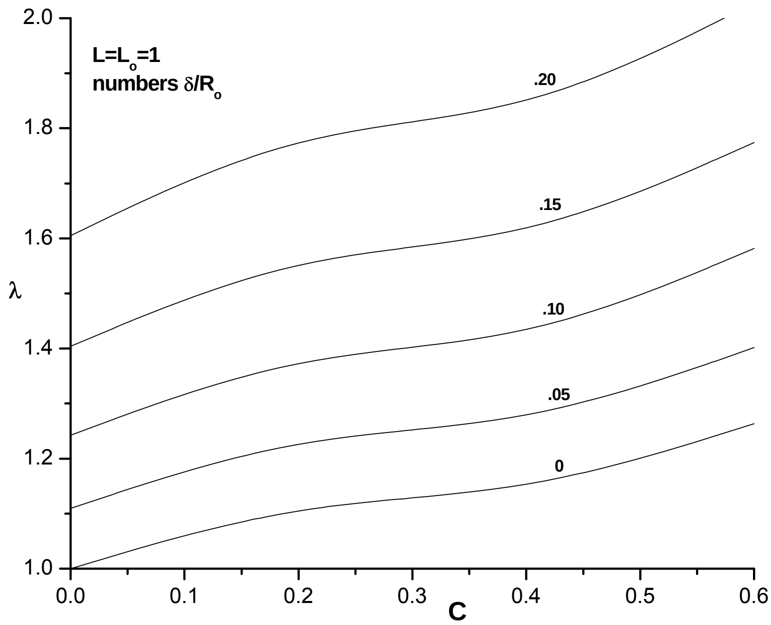

Fig. 4 Impedance, λ versus hematocrit, C for different stenosis height, δ/R_o.

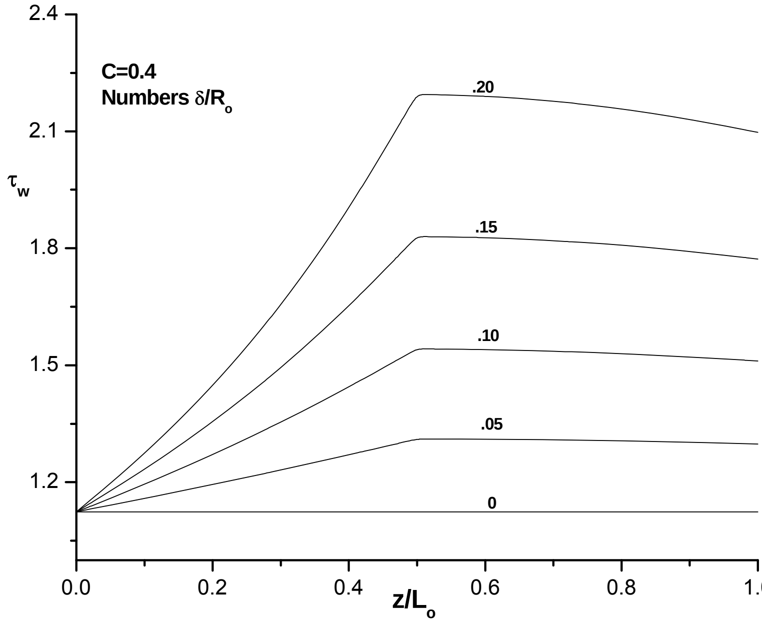

Fig. 5 Wall shear stress distribution, τ_w in the stenotic region for different stenosis height, δ/R_o.

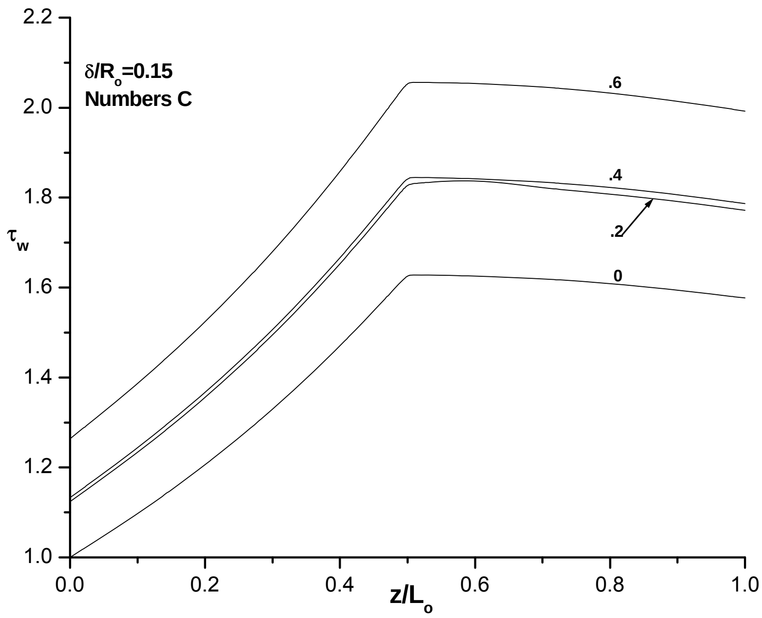

Fig. 6 Wall shear stress distribution, τ_w in the stenotic region for different hematocrit, C.

for any given stenosis height, δ/R_0 (Fig.6). It is important to note here that the blood flow characteristic, τ_w increases rapidly in the upstream of the stenosis throat (located at $d+L_0/2$) and achieves its peak magnitude at the throat. The shear stress, τ_w asymptotically decreases in the downstream of the throat and achieves a very little lower magnitude than its value at the throat at the end point (i.e., at $z/_{Lo}$=1) of the constriction profile (Figs. 5 & 6). The shear stress at the stenosis throat, τ_s increases with the hematocrit, C for any given stenosis height, δ/R_0, and also τ_s increases with the stenosis height, for any given hematocrit, C (Fig. 7). The blood flow characteristic, τ_s increases steeply with the hematocrit, C for any stenosis height, δ/R_0 (Fig.8). The variations in τ_s are found to be similar to that of the flow resistance, λ with respect to any parameter.

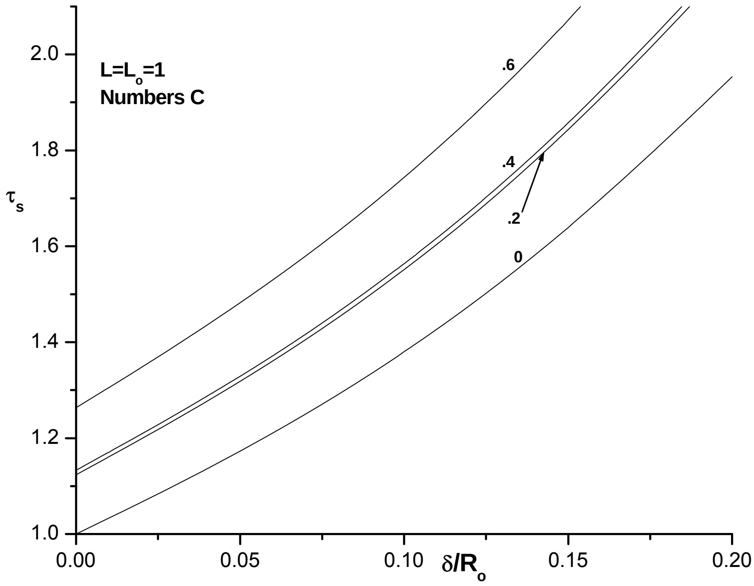

Fig. 7 Shear stress at stenosis throat, τ_s versus stenosis height, δ/R_o for different hematocrit, C.

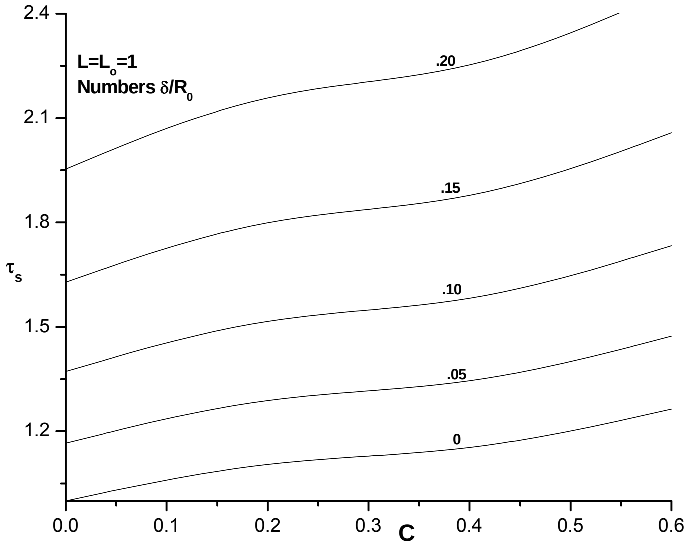

Fig. 8 Shear stress at stenosis throat, τ_s versus hemtocrit, C for different stenosis height, δ/R_o.

REFERENCES

1. Ahmed, P.S. and Giddens, D.P. Velocity measurements in steady flow through axisymmetric stenosis at moderate Reynolds number. J. Biomech. 16, 505-516, 1983.

2. Bandyopadhyay, S. and Layek, G. C. Numerical computation of pulsatile flow through a locally constricted channel. Commum. Nonlinear Sci. Numer. Simulat. 16, 252-265, 2011.

3. Bandyopadhyay, S. and Layek, G. C. Study of magnetodydrodynamic pulsatile flow in a constricted channel. Commum. Nonlinear Sci. Numer. Simulat. 17, 2434-2446, 2012.

4. Biswas, D. and Chakraborty, U. S. Pulsatile blood flow through a catherterized artery with a
axially nonsymmetrical stenosis. Appl. Math. Sci. 58(4), 2865-2880, 2010b.

5. Biswas, D. and Chakraborty, U. S. Two-layered pulsatile blood flow in stenosed artery with body acceleration and slip wall. Applc. Appl. Math. 5(2), 303-320, 2010a.
6. Caro, C.G., Pedley, T.J., Schroter, R.C. and Seed, W.A. The Mechanics of the circulation, Oxford Medical, N.Y., 1978.

7. Charm, S.E. and Kurland, G.S. Blood Flow and Micro Circulation. John Wiley, Newyork, 1974.

8. Charm, S.E. and Kurland, G.S. Blood Rheology in Cardiovascular Fluid Dynamics, Academic Press, London, 1965.

9. Cokelet, G.R. The Rheology of Human Blood: In Biomechanics, Prentice-Hall, Englewood Cliffs, N.J., 1972.

10. Deshpande, M.D. Giddens, D.P. and Mabon, R.F. Steady laminar flow through modeled vascular stenoses. J. Biomech. 9, 165-174, 1976.

11. Haynes, R.H. Physical Basis on dependence of blood viscosity on tube radius. Am. J. Physiol. 198, 1193-1205, 1960.

12. Hershey, D., Byrnes R.E., and Roam, A.M. Blood Rheology: Temperature dependent of the Power Law Model, presented at the A.I.C.H. Meeting, Boston, 1964.

13. Huckaba, C.E. and Hahu A.W. A generalized approach to the modeling of arterial blood flow. J. Appl. Physiol. 27, 27-34, 1968.

14. Ku, D.N. Blood flow in arteries. Ann. Rev. Fluid Mech. 29, 399-434, 1997.

15. Mann, F.C., Herrick, J.F., Essex, H.E. and Blades, E.J. Effects on blood flow of

decreasing the lumen of blood vessel. Surgery 4, 249-252, 1938.

16. Medhavi, A. On macroscopic two-phase arterial blood flow through an overlapping stenosis. E-Journal of Science and Technology 6, 19-31, 2011.
17. Mekheimer, Kh. S. and El-Cot, M. A.. Magnetic field and hall current on blood flow through stenotic artery. Appl. Math. And Mech. 29, 1-12, 2008.

18. Mekheimer, Kh. S., Harun, M. H. and Elkot, M. A. Induced magnetic field influences on blood flow through an anisotropically tapered elastic arteries with overlapping stenosis in an annulus. Can. J. Phys. 89, 210-212, 2011.
19. Merrill F.W., Benis, A.M., Gilliland. E.R., Sherwood, T.K. and Salzman, E.W. Pressure-Flow Relations of Human Blood in Hallow Fibers at Low Flow Rates. J. Appl. Physiol. 20, 954-967. 1965.

20. Mishra, B.K. and Verma, N. Effect of porous parameter and stenosis on wall shear stress for the flow of blood in human body. Res. J. Medicine and Medical Sciences 2, 98-101, 2007.

21. Mishra, S. and Siddiqui, S. U. A Mathematical model for blood flow and diffusion through stenotic capillary-tissue exchange system. E-J. Sci. & Tech. 6, 1-17, 2011.

22. Nadeem, S., Akbar, N.S., Hendi, A.A. and Hayat, T. Power law fluid model for blood flow through a tapered artery with a stenosis. Appl. Math. Comput. 217, 7108-7116, 2011.

23. Ponalagusamy, R. and Selvi, R. T. Blood flow through stenosed arteries: new formula for computing peripheral layer thickness. Int. J. Bio-Sci. & Bio-Tech. 3, 27-37, 2011.

24. Ponalagusamy, R. Blood flow through an artery with mild stenosis: A two layered model, different shapes of stenosis and slip velocity at the wall. J Appl. Sci. 7(7), 1071-1077, 2007.

25. Sankar, D. S. and Lee, U. Mathematical modeling of pulsatile flow of non-Newtonian fluid in stenosed arteries. Commum. Nonlinear Sci. Numer. Simulat. 14, 2971-2981, 2009.
26. Sarkar, A. and Jayaraman, G. Correction to flow rate-pressure drop in coronary angioplasty: steady streaming effect. J. Biomech. 31, 781-791, 1998.

27. Singh, B., Joshi, P. and Joshi, B.K. Blood flow through an artery having radially non-summetric mild stenosis. Appl. Math. Sci. 4(22), 1065-1072, 2010.
28. Srivastava, L. M. and Srivastava, V. P. Peristaltic transport of a particle-fluid suspension. Trans. ASME J. Biomech. Engng. 111, 157-165, 1989.
29. Srivastava, L.M. and Srivastava, V.P. On Two-Phase Model of Pulsatile Blood Flow with Entrance Effects. Biorheol. 20, 761-777, 1983.
30. Srivastava, V. P., Rastogi, R. and Vishnoi, R. A two-layered suspension blood flow through an overlapping stenosis. Computers and Mathematics with Applications 60, 432-441, 2010.

31. Srivastava, V.P. A Theoretical model for blood flow in small vessels. Applc. Appl. Maths. 2, 51-65, 2007.

32. Srivastava, V.P. and Rastogi, Rati. Effects of hematocrit on impedance and shear stress during stenosed artery catheterization. Applc. Appl. Math. 4, 98-113, 2009.

33. Srivastava, V.P. and Srivastava, Rashmi. Particulate suspension blood flow through a narrow catheterized artery. Comput. Math. Applic.58, 227-234, 2009.

34. Srivastava, V.P. Arterial blood flow through a non-symmetrical stenosis with applications. Jpn. J. Appl. Phys. 34, 6539-6545, 1995.

35. Srivastava, V.P., Vishnoi, R. and Sinha, P. response of a composite stenosis to non-Newtonian blood flow in arteries. E-J. Sci. & Tech. 7(2), 61-70, 2012.

36. Young, D.F. and Tsai, F.Y. Flow characteristics in models of arterial stenosis-I. Steady Flow. J. Biomech. 6, 395-410, 1973.

37. Young, D.F. Effects of a time-dependent stenosis of flow through a tube. J. Eng. Ind. 90, 248-254, 1968.

38. Young, D.F. Fluid mechanics of arterial stenosis. J. Biomech. Eng. 101, 157-175, 1979.