RESPONSE OF A COMPOSITE STENOSIS TO NON-NEWTONIAN BLOOD IN ARTERIES

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Abstract

The problem of blood flow through a composite stenosis in arteries has been investigated in the present work. To account for the non-Newtonian behavior, blood has been represented by a power-law fluid. The expression for the flow characteristics, namely, the impedance, the wall shear stress, the shear stress at the stenosis throat has been derived. We present some results concerning the dependence of these quantities on the geometrical parameters.

Key Words: Power-law, composite stenosis, impedance, shear stress, stenosis throat.

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INTRODUCTION

Cardiovascular diseases are known to be responsible in most of the cases of deaths and stenosis or arteriosclerosis (narrowing of any body passage, tube or orifice), is one of the common diseases. It is an unnatural and abnormal growth in the lumen of arterial wall segments, develops at various location of the cardiovascular system under the diseased conditions. Although the etiology of the initiation of the stenosis is not well understood, it has been suggested that the deposit of cholesterol on the arterial wall and proliferation of connective tissue may be responsible for the development of the disease. Regardless of the cause, it is well established that once the constriction has developed, it significantly affects the supply of the blood in arteries which may results into serious consequences (myocardial infraction, angina pectoris, cerebral strokes, etc.). The flow accelerates and consequently the velocity gradient near the wall region is steeper due to the increased core velocity resulting in relatively large shear stress on the wall even for a mild stenosis in the region of narrowing arterial constriction. With the knowledge that hemodynamic factors play an important role in the genesis and the proliferation of stenosis, this Area of research has attracted the attention of investigators. Since the first investigation of Mann et al. (1938), a large number of studies including the important contributions of Young(1968,1979), Young and Tsai (1973), Caro et al. (1978), Shukla et al.(1980), Ahmed and Giddens (1983), Sarkar and Jayaraman (1998), Pralhad and Schultz (2004), Jung et al. (2004), Liu et al. (2004) Srivastava and coworkers (1996, 2009, 2010a,b,c), Mishra et al. (2006), Misra and Verma (2007), Ponalagusamy (2007), Layek et al. (2005, 2009), Joshi et al. (2009), Mekheimer and El-Kot (2008), Tzirtzilakis (2008), Mandal and coworkers (2005, 2007a,b), Politis et al. (2007, 2008), Singh et al. (2010), Medhavi (2011) and many others; have been conducted in the literature in various context.

Being a suspension of corpuscles, blood behaves like a non-Newtonian fluid at low shear rate in small blood vessels (Hershey et al., 1964; Merill et al., 1965; Charm and Kurland, 1974). In particular, it has been pointed out that the flow behavior of blood in small diameter tubes (less than 0.2mm) at less than 20 sec^{-1} shear rate, can be reasonably represented by a power-law fluid (Hershey et al., 1964; Charm et al., 1965; Huckaba and Hahn, 1968). In addition, a survey of literature on stenotic development indicates that most of the studies have been conducted are concerned with a single symmetric or non-symmetric stenoses. However, the recent observations Joshi et al., 2009; Srivastava and Rastogi, 2010a) reveal that stenoses may develop in series (multiple stenoses) or may be of irregular shapes or overlapping or of composite in nature. The present research is therefore devoted to study the effects of a composite stenosis on blood flow characteristics assuming that the flowing blood is represented by a power law fluid.

FORMULATION OF THE PROBLEM

Consider the axisymmetric flow of blood through a composite stenosis in an artery of circular cross-section. The geometry of the composite stenosis, assumed to be manifested in the arterial wall segment is described (Fig.1) as

$$R(z)/R_0 = 1 - \frac{2\delta}{R_0 L_0}(z-d); \qquad d \le z \le d + L_0/2, \tag{1}$$

$$= 1 - \frac{\delta}{2R_0}\left\{1 + \cos\frac{2\pi}{L_0}(z - d - L_0/2)\right\}; \; d + L_0/2 \le z \le d + L_0, \tag{2}$$

$$= 1; \text{ otherwise}, \tag{3}$$

where $R \cong R(z)$ and R_0 are respectively, the radius of the artery with and without stenosis, L_0 is the length of the stenosis and d indicates its location, δ is the maximum projection in the lumen

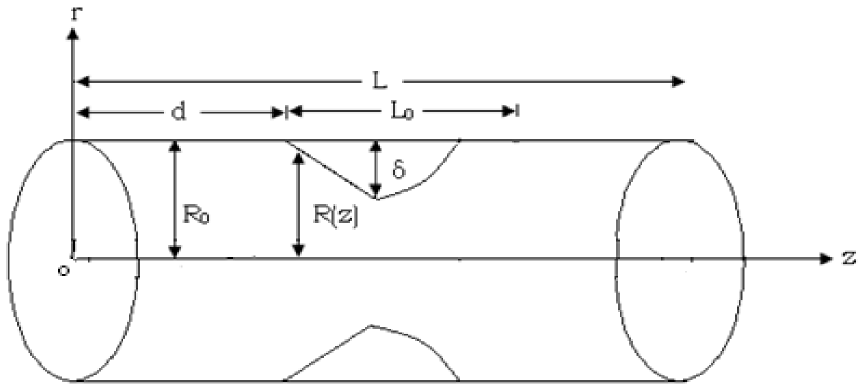

Fig. 1 Geometry of a composite stenosis in an artery.

located at $z=d+L_0/2$.

Blood is assumed to be represented by a non-Newtonion (power-law) fluid. Following the reports (Young, 1968; Srivastava, 1995) and considering laminar, steady, one dimensional flow of blood in an artery, the general constitutive equation in a mild stenosis case $\delta \ll R_0$, may be written as

$$\frac{dp}{dz} = -\frac{1}{r}\frac{\partial}{\partial r}(r\,\tau), \tag{4}$$

where r is the radial coordinate measured normal to the tube axis and p is the pressure. The shear stress, τ for a power-law fluid is given by

$$-\frac{du}{dr} = \left(\frac{\tau}{m}\right)^{1/n} , \tag{5}$$

where u is the axial velocity of the fluid, m is the consistency, and n is the flow behavior index (when $n = 1$, then $m = \mu$ is the Newtonion viscosity of the fluid).

The boundary conditions are

$$\frac{\partial u}{\partial r} = 0 \text{ at } r = 0. \tag{7}$$

ANALYSIS

Using equation (5) into (4), the solution of equation (4) under the boundary conditions (6) and (7), yields the expression for velocity, u as

$$u = -\frac{n}{n+1}\left(-\frac{1}{2m}\frac{dp}{dz}\right)^{1/n}\left(R^{(n+1)/n} - r^{(n+1)/n}\right). \tag{8}$$

The volumetric flow rate Q is now calculated as

$$Q = 2\pi\int_0^R ru\, dr$$

$$= \frac{n\,\pi}{3n+1}\left(-\frac{1}{2m}\frac{dp}{dz}\right)^{1/n} R^{(3n+1)/n}, \tag{9}$$

which yields

$$\frac{dp}{dz} = -2m\left(\frac{3n+1}{n\,\pi}Q\right)^{n}\frac{1}{R^{3n+1}}. \tag{10}$$

The pressure drop, Δp $(= p \text{ at } z = 0 \text{ and } - p \text{ at } z = L)$ across the stenosis in the tube of length, L is calculated as

$$\Delta p = \int_0^L\left(-\frac{dp}{dz}\right)dz$$

$$= \frac{2m}{R_0^{3n+1}}\left(\frac{3n+1}{n\,\pi}Q\right)^{n}\psi, \tag{11}$$

where

$$\psi = \int_0^d [\phi(z)]_{R/R_0=1}dz + \int_d^{d+L_0/2}[\phi(z)]_{R/R_0 \text{ from (1)}}dz + \int_{d+L_0/2}^{d+L_0}[\phi(z)]_{R/R_0 \text{ from (2)}}dz + \int_{d+L_0}^{L}[\phi(z)]_{R/R_0=1}dz, \tag{12}$$

with $\phi(z) = \dfrac{1}{(R/R_0)^{3n+1}}$.

The analytical evaluation of the first, second and the fourth integrals are straight forward where as the computation of third integral in the closed form in the expression for ψ equation (12) is almost a formidable task and thus will be evaluated numerically. Using the definitions from the published literature (Srivastava,1996), the expressions for the impedance (flow resistance), $\bar{\lambda}$, the wall shear stress in the stenotic region, $\bar{\tau}_w$ and the shear stress at stenosis throat, $\bar{\tau}_w$ are given as

$$\bar{\lambda} = \Delta p/Q, \tag{13}$$

$$\bar{\tau}_w = (-R/2)dp/dz, \tag{14}$$

$$\bar{\tau}_w = (-R/2)dp/dz, \tag{15}$$

Following now the reports of Young(1968) and Srivastava et.al.(2010a), the expression for impedance, λ, the wall shear stress, τ_w and shear stress at stenosis throat, τ_s, in their non-dimensional form are derived as

$$\lambda = (1 - L_0/L) + \frac{R_0 L_0}{6n\delta L}\left\{1 - \frac{1}{(1-\delta/R_0)^{3n}}\right\} + \frac{L_0}{2\pi L}\int_0^{\pi}\frac{d\beta}{(a + b\cos\beta)^{3n+1}}, \tag{16}$$

$$\tau_w = \frac{1}{(R/R_0)^{3n}}, \tag{17}$$

$$\tau_s = \frac{1}{(1-\delta/R_0)^{3n}}, \tag{18}$$

where $a = 1 - \delta/R_0$ and $b = \delta/2R_0$, $\beta = \pi - (2\pi/L_0)(z - d - L_0/2)$, $\lambda = \lambda/\lambda_0$, $\tau = \tau/\tau_0$, λ_0 and τ_0 are the impedance and the shear stress for a power-law fluid in the normal artery (i.e., nostenosis) and are given by

$$\lambda_0 = \frac{2mL}{QR_0^{3n+1}}\left(\frac{3n+1}{n\pi}Q\right)^n \text{ and } \tau_0 = \frac{m}{R_0^{3n+1}}\left(\frac{3n+1}{n\pi}Q\right)^n.$$

It is worth to mention here that results obtained above reduce to the corresponding results for a Newtonian fluid (Young, 1968) for the artery with a composite stenosis.

RESULTS AND DISCUSSION

To discuss the results obtained above quantitatively, computer codes are developed to evaluate the analytical results obtained in equations (16)-(18). The various parameter values are selected (Young, 1968; Srivastava, 1996; Hershey et.al., 1964; Charm and Kurland, 1974) as

$$R_0(cm) = 0.01\ L_0/L = 1, 0.5, 0.1 : n = 1, 2/3, 1/3;\quad \delta/R_0 = 0, 0.05, 0.10, 0.15, 0.20.$$

It is to note here that the present study corresponds to a Newtonian fluid case and to the flow in normal artery for parameter value $n = 1$ and $\delta/R_0 = 0$, respectively.

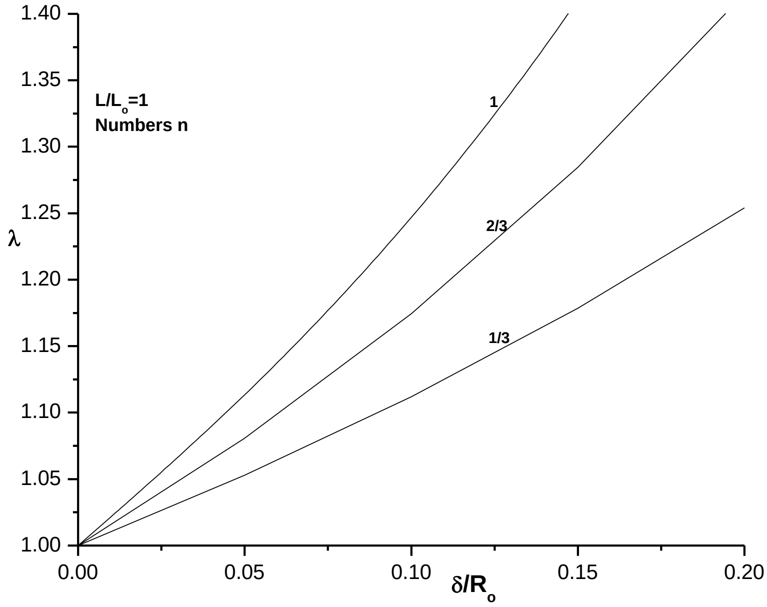

Fig.2 Impedance, λ versus stenosis height, δ/R_o for different, n.

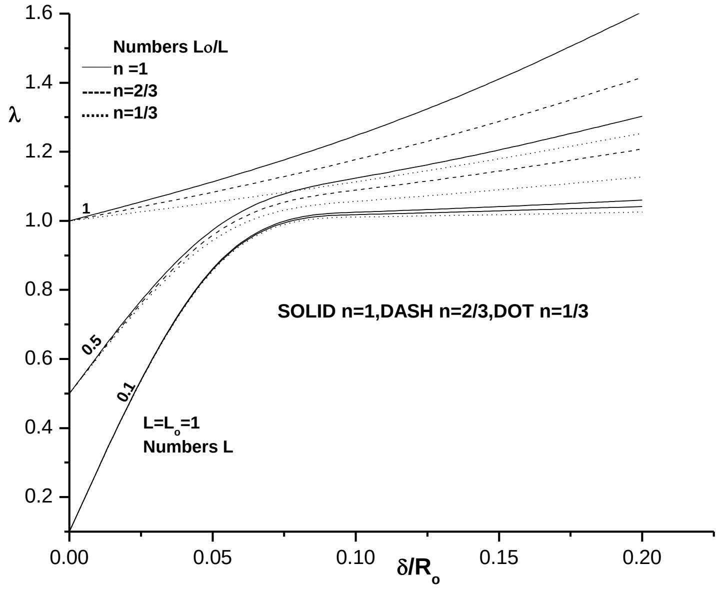

Fig.3 Impedance, λ versus stenosis height, δ/R_o for different, L.

The impedance (resistance to flow), λ with the stenosis height, δ/R_0 for any given value of the flow behavior index, n (Fig.2). The impedance, λ decreases with the stenosis length, L

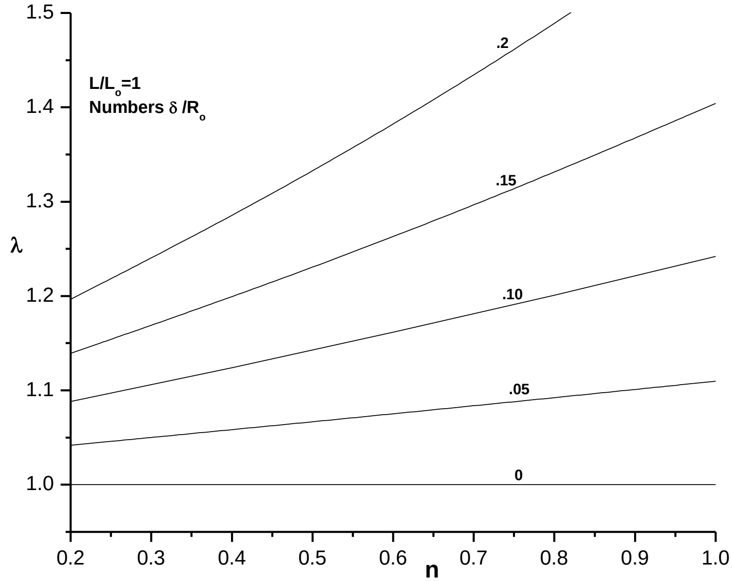

Fig.4 Impedance, λ versus stenosis height, δ/Ro for different, n.

for any given value of the flow behavior index, n and the stenosis height, δ/R_0 (Fig.3). We observe that for any given stenosis height, δ/R_0, the flow resistance, λ assumes the maximum magnitude in Newtonion fluid (n=1) analysis (Fig.4).

The wall shear stress in the stenotic region, τ_w increases with the stenosis height, δ/R_0 at any axial distance, z/L_0 (Fig.5). At any axial distance, z/L_0 in the stenotic region, the flow characteristic, τ_w decreases with the flow behavior index, n for a given height, δ/R_0 (Fig.6). The wall shear stress in the stenotic region, τ_w increases steeply in the upstream of the stenosis throat located at $z=d+L_0/2$ assumes its peak magnitude at the throat and then decreases

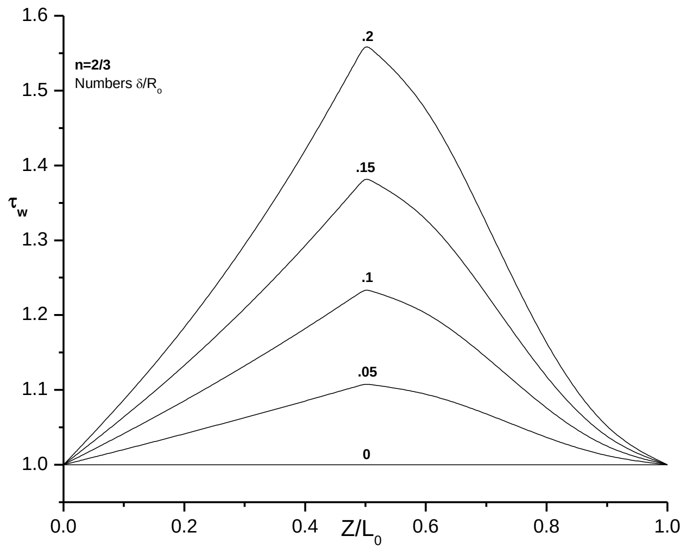

Fig.5 Wall shear stress, τ_w versus z/L_0 in the stenotic region for different stenosis height, δ/R_o.

rapidly in the downstream of the throat and achieves the same value at the end of the constriction profile at z = d+L_0 as its approached value at z = d. One notices that there exists almost a linear relationship between the blood flow characteristic, τ_w and the axial distance, z/L_0 in the upstream of the stenosis throat, however, the relationship between τ_w and z/L_0 is observed to be clearly a nonlinear for any given set of parameters (Fig.5 and 6). The shear stress at the stenosis throat, τ_s decreases with the flow behavior index n for any given stenosis height, δ/R_0 (Fig.7). Numerical results reveal that the blood flow characteristic, τ_s increases with the stenosis size (height and length). The nature of variations of the shear stress at stenosis throat, τ_s are similar to that of the impedance, λ with respect to any parameter (Fig.2 and 7).

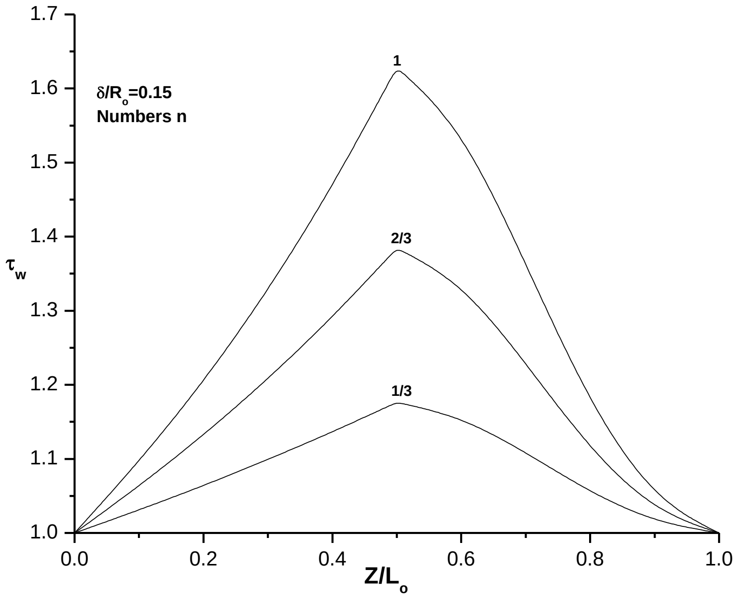

Fig.6 Wall shear stress, τ_w versus z/L_0 in the stenotic region for different n.

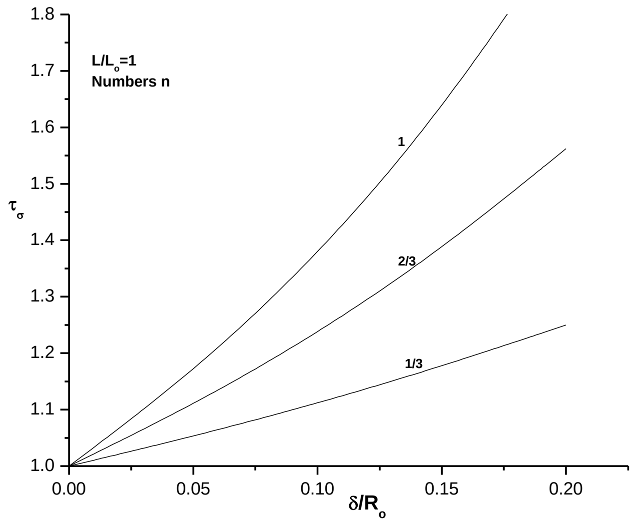

Fig.7 Shear stress at stenosis throat, τ_s versus stenosis height, δ/R_o for different, n.

CONCLUSIONS

A non-Newtonian (power-law) fluid has been used to discuss the response of a composite stenosis on the flow characteristics of blood. The flow resistance decreases with the non-Newtonian behavior of blood but increases with the stenosis size (height and length both). The wall shear stress at any axial distance in the stenotic region including at the setnosis throat possesses characteristics similar to that of the flow resistance with respect to any parameter.

REFERENCES

1. Ahmed, A. S. and Giddens, D. P. Velocity measurements in steady flow through axisymmetric stenosis at moderate Reynolds number. Journal of Biomechanics, 16, 505-516, 1983.
2. Caro, C. G. Pedley, T. J., Schroter, R.C. and Seed, W. A. The Mechanics of the Circulation. Oxford Medical, N. Y, 1978.
3. Charm, S. E. and Kurland, G. S. Blood Flow and Microcirculation. John Wiley, N. Y. 1974.
4. Chein, S., Usmani, S., Taylor, H. M., Lundberg, J. L. and Gregerson, M. T. Effects of hematocrit and plasma proteins on human blood rheology at low shear rates. J. Appl. Physiol.,21, pp. 81-87, 1965.
5. Hersey, D., Byrnes, R. E. and Roam, A. M. Blood rheology: temperature dependent of the power-law model, Presented at the A. I. C. H. Meeting, Boston, 1964.
6. Huckaba, C. E. and Hahn, A. W.A generalized approach to the modeling of arterial blood flow. Bull. Math. Biophys. 30, 645-662, 1968.
7. Joshi, P., Pathak, A. and Joshi, B.K. Two layered model of blood flow through composite stenosed artery. Applications and Applied Mathematics, 4(2), 343-354, 2009.
8. Jung. H. Choi, J.W. and Park, C.G. Asymmetric flows of non-Newtonian fluids in symmetric stenosed artery. Korea-Aust. Rheol. Journal, 16, 101-108, 2004.
9. Layek, G.C., Mukhopadhyay, S. and Gorla, R.S.R. Unsteady viscous flow with variable viscosity in a vascular tube with an overlapping constriction. Int. J. Engg. Sci. 47, 649-659, 2009.
10. Layek, G.C., Mukhopadhyay, S. and Samad, Sk. A. Oscillatory flow in a tube with multiple constrictions, Int. J. Fluid Mech. Res. 32, 402-419, 2005.
11. Liu, G.T., Wang, X.J., Ai, B.Q. and Liu, L.G. Numerical study of pulsating flow through a tapered artery with stenosis. Chin. Journal Phys., 42, 401-409, 2004.
12. Mandal, P.K. An unsteady analysis of non-Newtonian blood flow through tapered arteries with a stenosis. Int. J. of Nonlinear Mechanics. 40, 51–164, 2005.
13. Mandal, P.K., Chakravarty, S. and Mandal, A. Numerical study on the unsteady flow of non-Newtonian fluid through diffrerently shaped arterial stenosis. Int. J. Comput. Math. 84, 1059-1077, 2007a.
14. Mandal, P.K., Chakravarty, S., Mandal, A. and Amin, A. Effect of body acceleration on unsteady pulsatile flow of non-Newtonian fluid through a stenosed artery. Appl. Math. Comput. 189, 766-779, 2007b.
15. Mann, F. C., Herrick, J. F., Essex, H. E. and Blades, E. J. Effects on blood flow of decreasing the lumen of blood vessels. Surgery 4, 249-252, 1938.
16. Medhavi, A. On macroscopic two-phase arterial blood flow through an overlapping stenosis. E-Journal of Science and Technology 6, 19-31, 2011.

17. Mekheimer, Kh. S. and El-Kot. Magnetic field and hall currents influences on blood flow through a stenotic arteries, Applied Mathematics and Mechanics, 29, 1-12, 2008.
18. Merrill. F. W., Benis, A. M. Gilliland, E. R. Sherwood, T. K. and Salzman, E. W. Pressure-flow relations of human blood in hollow fibers at low shear rates. J. Appl. Physiol., 20, 954-967, 1965.
19. Mishra, B. K. and Verma, N. Effects of porous parameter and stenosis on the wall shear stress for the flow of blood in human body. Res. J. medicine and Medical Sciences, 2, 98-101, 2007.
20. Misra, J.C. and Shit, G.C. Blood flow through arteries in a pathological state: A theoretical study. Int. J. Engg. Sci. 44, 662-671, 2006.
21. Politis, A. K., Stavropoulos, G. P., Christolis, M. N., Panagopoulos, F. G., Vlachos, N. S., and Markatos, N. C. Numerical modeling of simulated blood flow in idealized composite arterial coronary grafts: Steady state simulations. J. Biomech., 40(5), 1125–1136, 2007.
22. Politis, A. K., Stavropoulos, G. P., Christolis, M. N. Panagopoulos, F. G. , Vlachos, N. S. and Markatos, N. C. Numerical modeling of simulated blood flow in idealized composite arterial coronary grafts: Transient flow. J. Biomechanics. 41(1), 25-39, 2008.
23. Ponalagusamy, R. Blood flow through an artery with mild stenosis: A two layered model, different shapes of stenosis and slip velocity at the wall. J Appl. Sci. 7(7), 1071-1077, 2007.
24. Pralhad, R.N. and Schultz, D.H. Modeling of arterial stenosis and its applications to blood diseases. Math. Biosci., 190, 203-220, 2004.
25. Sarkar, A. and Jayaraman, G. Corretion to flow rate-pressure drop in coronary angioplasty; steady streaming effect. Journal of Biomechanics, 31, 781-791, 1998.
26. Shukla, J. B., Parihar, R. S. and Gupta, S. P. Effects of peripheral layer viscosity on blood flow thorough the arterywith mild stenosis. Bull. Math. Biol. 42, 797-805, 1980.
27. Singh, B., Joshi, P. and Joshi, B.K. Blood flow through an artery having radially non-summetric mild stenosis. Appl. Math. Sci. 4(22), 1065-1072, 2010.
28. Srivastava, V. P. and Rastogi, R. Blood flow through stenosed catherterized artery: effects of hematocrit and stenosis shape. Comput. Math. Applc.59, 1377-1385, 2010a.
29. Srivastava, V. P. and Rastogi, R. Effects of hematocrit on impedance and shear stress during stenosed artery catheterization. Applications and Applied Mathematics, 4, 98-113, 2009.
30. Srivastava, V. P. Arterial blood flow through a nonsymmetrical stenosis with applications. Jpn. J. Appl. Phys. 34, 6539-6545, 1995.
31. Srivastava, V.P. Two-phase Model of Blood Flow Through Stenosed Tubes in the Presence of a Peripheral Layer: J. Biomech. 29, 1377-1382, 1996.
32. Srivastava, V.P., Vishnoi, Rochana and Sinha, Poonam. Particulate Suspension Blood Flow through a Stenosed Catheterized Artery. Applications and Applied Mathematics, 5, 1352-1368, 2010c.
33. Srivastava, V.P., Vishnoi, Rochana, Mishra, Shailesh, Sinha, Poonam. Blood flow through a composite stenosis in catheterized arteries. e- J. Sci. Tech., 5, 2010b.
34. Tzirtzilakis, E.E.Biomagnetic fluid flow in a channel with stenosis, Physica D. 237, 66-81, 2008.
35. Young, D. F. and Tsai F. Y. Flow characteristics in model of arterial stenosis – steady flow. Journal of Biomechanics, 6, 395-410, 1973.

36. Young, D. F. Effects of a time-dependent stenosis of flow through a tube. Journal of Eng. Ind., 90, 248-254, 1968.
37. Young, D. F. Fluid mechanics of arterial stenosis. J. Biomech. Eng. ASME. 101, 157-175, 1979.