RESPONSE TO BLOOD FLOW THROUGH AN OVERLAPPING STENOSIS IN CATHETERIZED ARTERIES

V. P. Srivastava, Rochana Vishnoi, Dharmendra Singh[1], Amit Medhavi[2], Shailesh Mishra

Department of Mathematics, Krishna Girls Engineering College, Kanpur-209217,India.
[1]Department of Mathematics, S.M.S. Institute of Technology,
Sultanpur Road, Lucknow, India.
[2]Department of Mechanical Engg. Kamla Nehru Institute of Technology,
Sultanpur-228 118, India.

Abstract

The present work concerns the effects on the flow characteristics of blood due to the presence of an overlapping stenosis in an artery assuming that the flowing blood is represented by a Newtonian fluid. The expressions for the blood flow characteristics-the impedance, the wall shear stress distribution in the stenotic region, shearing stress at the stenosis throats and at the stenosis critical height have been derived. Results obtained have been displayed graphically and discussed briefly.

Key words: Two-phase, hematocrit, impedance, shear stress, stenosis throat.

Introduction

Stenosis means narrowing of any body passage, tube or orifice, is an abnormal and unnatural growth in the arterial wall thickness that develops at the various locations of the cardiovascular system under diseased conditions and occasionally results in to serious consequences (cerebral strokes, myocardial infarction, angina pectoris, cardiac arrests, etc.). The disease seems to occur due to the deposits of the cholesterol, fatty substances, cellular waste products, calcium and fibrin in the inner lining of the of an artery. Irrespective of the cause, it is well known that once the constriction has developed, it brings about the significant changes in the flow field. With the advent of the discovery that the cardiovascular disease, stenosis is closely associated with the flow conditions and other hemodynamic factors, a large number of researchers including the important contributions of Mann (1938), Young (1968, 1979), Young and Tsai (1973), Caro et al. (1978), Shukla et al. (1980), Ahmed and Giddens (1983), Sarkar and Jayaraman (1998), Jung et al. (2004), Liu et al. (2004), Srivastava and coworkers (1996, 2009, 2010), Mishra and Shift (2006), Ponalagusamy (2007), Layek et al. (2005, 2009), Mekheimer and El-Kot (2008), Tzirtzilakis (2008), Mandal and coworkers (2005, 2007), Politis et al. (2007, 2008), Singh et al. (2010), Medhavi (2011), and many others have addressed the stenotic development problems under various flow conditions.

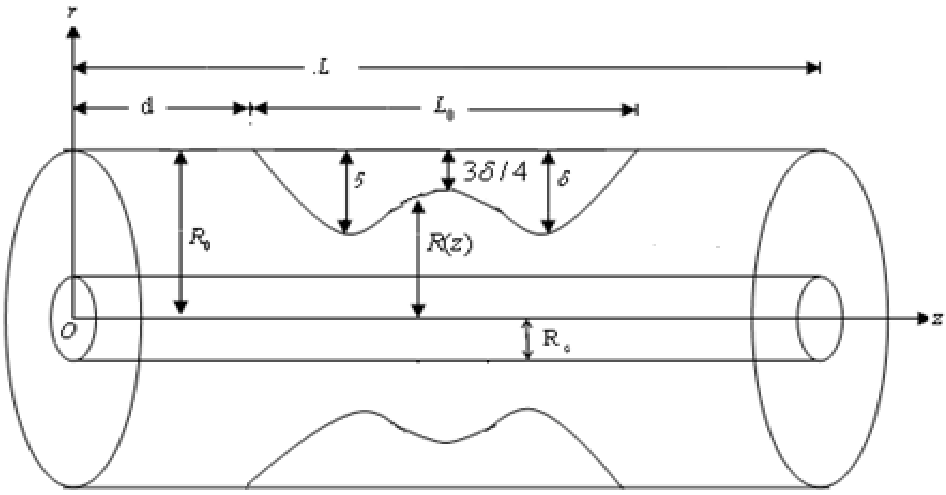
The use of catheters has become standard tool for diagnosis and treatment in modern medicine. When a catheter is inserted into the stenosed artery, the further increased impedance (resistance to flow) and shear stress will alter the flow field. A brief review of the literature on artery catheterization with and without stenosis has recently been presented by Srivastava and Srivastava (2009). A survey of the literature on stenotic development indicates that most of the studies in the literature are mainly concerned with single symmetric or non-symmetric stenoses. In a reatistic situation, however, the constrictions may develop in series (multiple stenoses), may be of irregular shapes, overlapping, composite in nature, etc. Chakravarty and Mandal (1994) studied analytically the effects of an overlapping stenosis on arterial flow problem of blood assuming the pressure variation only along the axis of the tube. Layek et al. (2009) investigated the effects of an overlapping stenosis on flow characteristics considering the pressure variation in both the radial and axial directions of the arterial segment under consideration. An effort is made in the present investigation to explore the effects of the inserted catheter in an artery with overlapping stenosis assuming that the flowing blood is represented by a Newtonian fluid.

Formulation of the Problem

Consider the axisymmetric flow of blood through an artery of circular cross section with an overlapping stenosis specified at the position as shown in Fig.1. The geometry of the stenosis which is assumed to be manifested in the arterial wall segment is described (Chakravarti and Mandal, 1994; Layek et al., 2009; Srivastava et al., 2010) as

$$\frac{R(z)}{R_0} = 1 - \frac{3}{2}\frac{\delta}{R_0 L_0^4}\left[11(z-d)L_0^3 - 47(z-d)^2 L_0^2 + 72(z-d)^3 L_0 - 36(z-d)^4\right],$$

$$= 1, \text{ otherwise,} \tag{1}$$



Fig, 1 Flow geometry of an overlapping stenosis in a catheterized artery.

where ($R(z)$, R_0) are the radius of the tube (with, without) stenosis, L_0 is the stenosis length, d indicates stenosis location, δ is the maximum height of the stenosis into the lumen, appears at the locations: $z = d+L_0/6$ and $d+5L_0/6$, z being the axial coordinate. The height of the stenosis at $z = d+L_0/2$, called critical height, is $3\delta/4$.

The flowing blood is assumed to be represented by a Newtonian fluid, using thus a continuum approach the equations governing the linear momentum and the conservation of mass for a Newtonian fluid are given by

$$\rho\left\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}\right\}=-\frac{\partial p}{\partial z}+\mu\nabla^2 u, \tag{2}$$

$$\rho\left\{\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial r}\right\}=-\frac{\partial p}{\partial r}+\mu(\nabla^2-\frac{1}{r^2})\,v\ , \tag{3}$$

$$\frac{\partial v}{\partial r}+\frac{v}{r}+\frac{\partial u}{\partial z}=0, \tag{4}$$

where $\nabla^2=\partial/\partial r^2+(1/r)(\partial/\partial r)+\partial^2/\partial z^2$ is a two-dimensional Laplacian operator, r is the radial coordinate measured in the direction normal to the tube axis, (u, v) denotes the (axial, radial) components of velocity of the fluid, p is the pressure, ρ and μ are respectively the fluid density and the viscosity.

Due to the non-linearity of convective acceleration terms, to obtain the solution of equations (2)–(4) a formidable task. Depending therefore on the size of the stenosis, however, certain terms in these equations are of less importance than other (Young, 1968). Considering thus the case of a mild stenosis $(\delta/R_0 \ll 1)$, the general constitutive equations (2)-(4) in the case of an axisymmetric, laminar, steady one-dimensional flow of blood in an artery reduce (Young, 1968; Srivastava and Rastogi, 2009) to

$$\frac{dp}{dz}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right), \tag{5}$$

$$\frac{dp}{dr}=0. \tag{6}$$

The boundary conditions are

$$u=0 \ \text{ at } \ r=R(z), \tag{7}$$

$$u=0 \ \text{ at } \ r=R_c. \tag{8}$$

Conditions (5) and (6) are the standard no slip conditions at the artery and catheter walls, respectively.

Analysis

The expression for the velocity obtained as the solution of equations (5) and (6), subject to the boundary conditions (7) and (8), is given as

$$u=-\frac{R_0^2}{4\mu}\frac{dp}{dz}\left[(R/R_0)^2-(r/R_0)^2+\frac{\{(R/R_0)^2-(R_c/R_0)^2\}}{\log(R/R_c)}\log\left(\frac{r}{R}\right)\right] \tag{9}$$

The flow flux, Q is thus calculated as

$$Q = 2\pi \int_{Rc}^{R} r\, u\, dr$$
$$= -\frac{\pi R_0^4 \left\{(R/R_0)^2 - \varepsilon^2\right\}}{8\mu} \frac{dp}{dz}\left[(R/R_0)^2 + \varepsilon^2 - \frac{\left\{(R/R_0)^2 - \varepsilon^2\right\}}{\log\{(R/R_0)/\varepsilon\}}\right] \qquad (10)$$

where $\varepsilon = R_c/R_0$. From equation (10), one now obtains

$$-\frac{dp}{dz} = \frac{8\mu\, Q}{\pi R_0^4}\varphi(z) \qquad (11)$$

with $\varphi(z) = 1/h(z),\ h(z) = \left\{(R/R_0)^2 - \varepsilon^2\right\}\left[(R/R_0)^2 + \varepsilon^2 - \frac{\left\{(R/R_0)^2 - \varepsilon^2\right\}}{\log\{(R/R_0)/\varepsilon\}}\right]$.

The pressure drop, $\Delta p\ (= p \text{ at } z = 0, - p \text{ at } z = L)$ across the stenosis in the tube of length, L is obtained as

$$\Delta p = \int_0^L \left(-\frac{dp}{dz}\right) dz$$
$$= \frac{8\mu Q}{\pi R_0^4}\psi, \qquad (12)$$

where $$\psi = \int_0^d [\varphi(z)]_{R/R_0=1} dz + \int_d^{d+L_0} [\varphi(z)]_{R/R_0 \text{ from } (1)} dz + \int_{d+L_0}^{L} [\varphi(z)]_{R/R_0=1} dz.$$

The first and the third integrals in the expression for ψ obtained above are straight forward whereas the analytical evaluation of second integral is almost a formidable task and therefore shall be evaluated numerically. Following now the definitions given in Srivastava and Rastogi (2009), one derives the expressions for the impedance (flow resistance), λ, the wall shear stress distribution in the stenotic region, τ_w and shear stress at the stenosis throat, τ_s in their non-dimensional form as

$$\lambda = \left\{\frac{1 - L_0/L}{\Omega} + \frac{1}{L}\int_d^{d+L_0} \frac{dz}{\left\{(R/R_0)^2 - \varepsilon^2\right\}\left\{(R/R_0)^2 + \varepsilon^2 - \frac{(R/R_0)^2 - \varepsilon^2}{\log[(R/R_0)/\varepsilon]}\right\}}\right\}, \qquad (13)$$

$$\tau_w = \frac{(R/R_0)}{\left\{(R/R_0)^2 - \varepsilon^2\right\}\left\{(R/R_0)^2 + \varepsilon^2 - \left(\left[(R/R_0)^2 - \varepsilon^2\right]/\log[(R/R_0)/\varepsilon]\right)\right\}}, \qquad (14)$$

$$\tau_s = \frac{(1 - 5\delta/4R_0)}{\left\{(1 - 5\delta/4R_0)^2 - \varepsilon^2\right\}\left\{(1 - 5\delta/4R_0)^2 + \varepsilon^2 - \left(\left[(1 - 5\delta/4R_0)^2 - \varepsilon^2\right]/\log[(1 - 5\delta/4R_0)/\varepsilon]\right)\right\}}, \qquad (15)$$

$$\tau_c = \frac{(1-3\delta/4R_0)}{\{(1-3\delta/4R_0)^2-\varepsilon^2\}\{(1-3\delta/4R_0)^2+\varepsilon^2-\left(\left[(1-3\delta/4R_0)^2-\varepsilon^2\right]/\log[(1-3\delta/4R_0)/\varepsilon]\right)\}}, \tag{16}$$

where $\lambda = \bar{\lambda}/\lambda_0,\ (\tau_w, \tau_s, \tau_c) = (\bar{\tau}_w, \bar{\tau}_s, \bar{\tau}_c)/\tau_0,$

$$\lambda = 8\mu L/\pi R_0^4, \quad \tau_0 = 4\mu Q/\pi R_0^3, \ \Omega = (1-\varepsilon^2)\{1+\varepsilon^2+(1-\varepsilon^2)/\log\varepsilon\},$$

λ_0 and τ_0 are respectively the resistive impedance and wall shear stress for an uncatheterized normal artery

(no stenosis). In the absence of the catheter (i.e., under the limit $\varepsilon \to 0$), one derives the expressions for λ, τ_w,

τ_s and τ_c, respectively through an overlapping stenosis as

$$\lambda = 1 - \frac{L_0}{L} + \frac{1}{L}\int_{d}^{d+L_0} \frac{dz}{(R/R_0)^4}, \tag{17}$$

$$\tau_w = \frac{1}{(R/R_0)^3}, \tag{18}$$

$$\tau_s = \frac{1}{(1-1.25\delta.2_0)^3}, \tag{19}$$

$$\tau_c = \frac{1}{(1-0.75\delta.7_0)^3}. \tag{20}$$

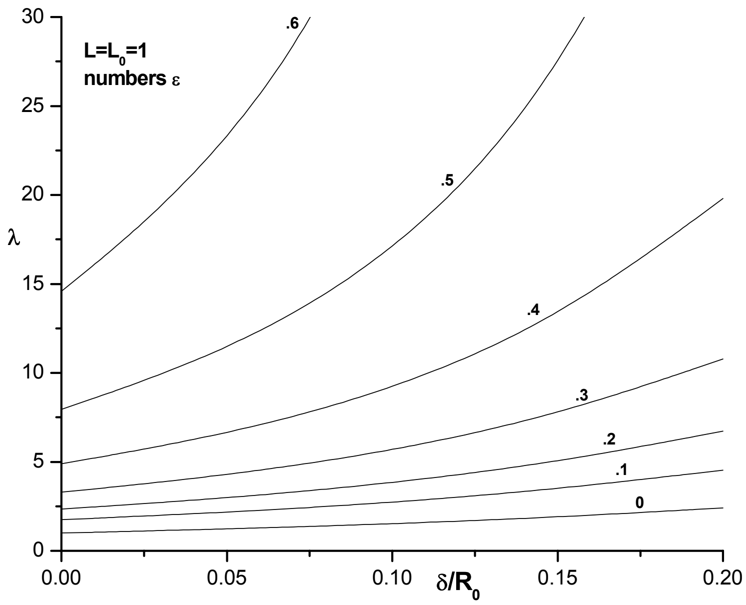


FIG. 2 Impedance, λ versus δ/R_0 for different ε.

Numerical Results and Discussion

To discuss the results of the study quantitatively, computer codes are now developed for the numerical evaluations of the analytical results obtained in equations (11)-(13). The various parameter values are selected from Young (1968), Srivastava and coworkers (2009, 2010) as: L_0 (cm) = 1; L (cm) = 1, 2, 5; ε (non-dimensional catheter radius) = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6; δ/R_0 (non-dimensional stenosis height) = 0, 0.05, 0.10, 0.15, 0.20. It is to note here that the present study corresponds to the flow in uncatheterized and normal (no stenosis) artery for parameter values ε=0 and δ/R_0 = 0, respectively.

The impedance, λ increases with the catheter size, ε for any given stenosis height, δ/R_0 and also increases with stenosis height, δ/R_0 for any given catheter size, ε (Fig.2). One notices that for any given stenosis height, a significant increase in the magnitude of the flow resistance, λ occurs for any small increase in the catheter size, ε (Fig.2). Numerical results reveal that for any given set of other parameters, the impedance, λ decreases with increasing the tube length, L which interns implies that the flow resistance, λ increases with the stenosis length, L_0. The flow resistance, λ steeply increases with the catheter size, ε ($\leq$ 0.3) but increases rapidly with increasing the catheter size, ε and depending on the height of the stenosis, attains a very high asymptotic magnitude with increasing the catheter size, ε (Fig.3). The high asymptotic magnitude of λ occurs for δ/R_0 = 0.1(19% stenosis), 0.15 (28% stenosis) and 0.2 (38% stenosis) at catheter size, ε = 5.5, 4.5 and 4.0, respectively.

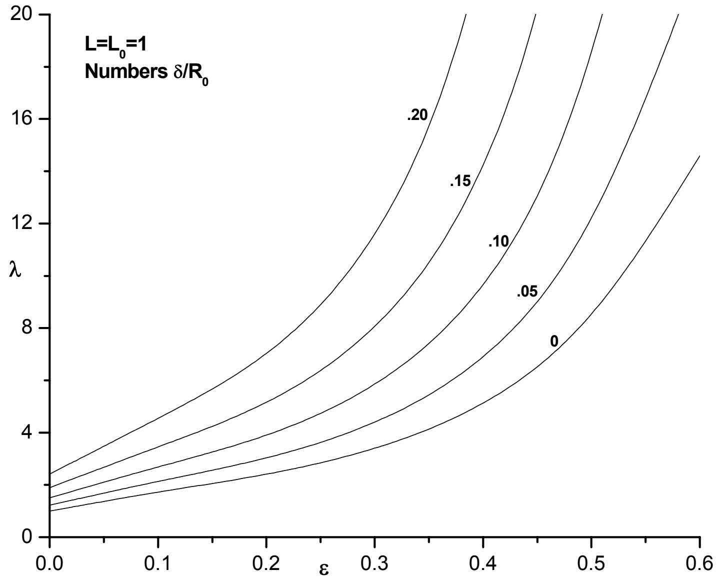


FIG. 3 Impedance, λ versus ε for different δ/R_0.

The wall shear stress in the stenotic region, τ_w increases rapidly with the axial distance z/ L_0 in the upstream of the stenosis first throat and attains its peak value at the first throat (i.e., at z/ L_0 =1/6). It then decreases steeply in the down stream of the first throat to its magnitude at stenosis critical height located at z/L_0=1/2. τ_w further increases steeply in the up stream of the stenosis second throat (at $z/L_0=5/6$) and attains its peak value at the second throat (same as at the first throat). The flow characteristic τ_w then decreases in the down stream of the stenosis second throat and achieves its approached magnitude (at $z/L_0=0$) at the end point of the constriction profile at $z/L_0=1$ (Fig.4). Numerical results also indicate that wall shear stress, τ_w increases with the catheter size, ε for any given stenosis height, δ/R_0 at any axial distance z/L_0.

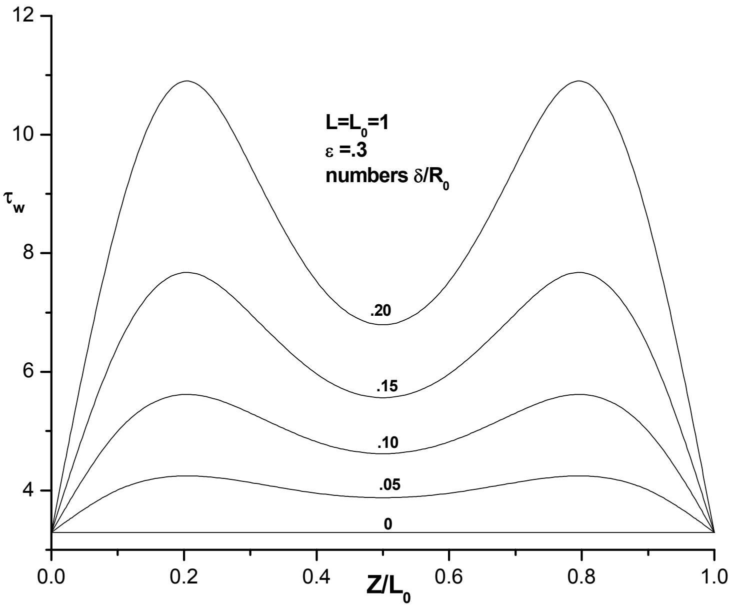


Fig. 4 Wall shear stress, τ_w in stenotic region for different δ/R_0.

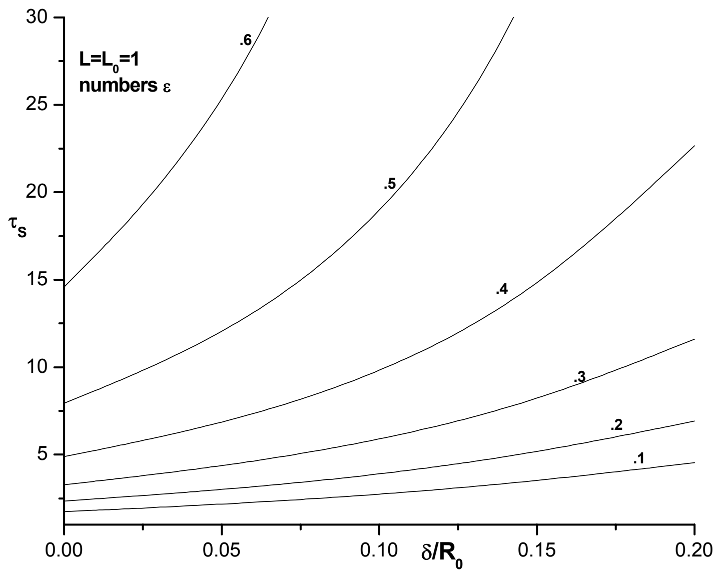


FIG. 5 Shear stress at stenosis throats, τ_s versus δ/R_0 for different ε.

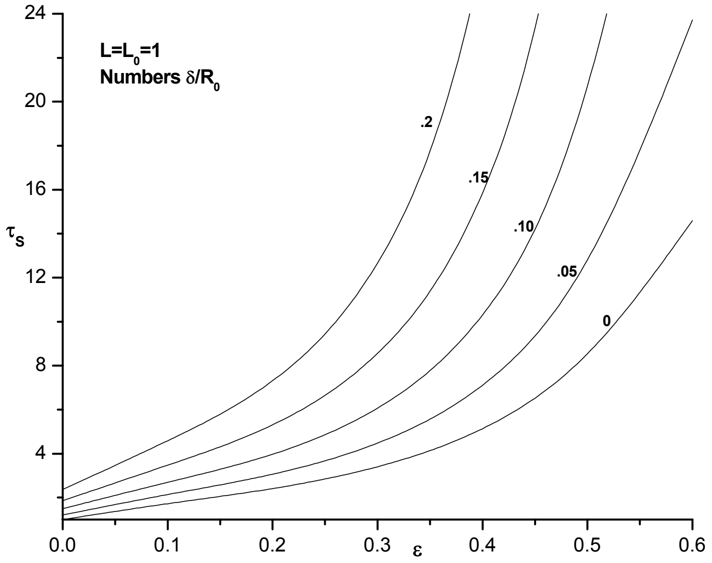


FIG. 6 Shear stress at stenosis throats, τ_s versus ε for different δ/R_0.

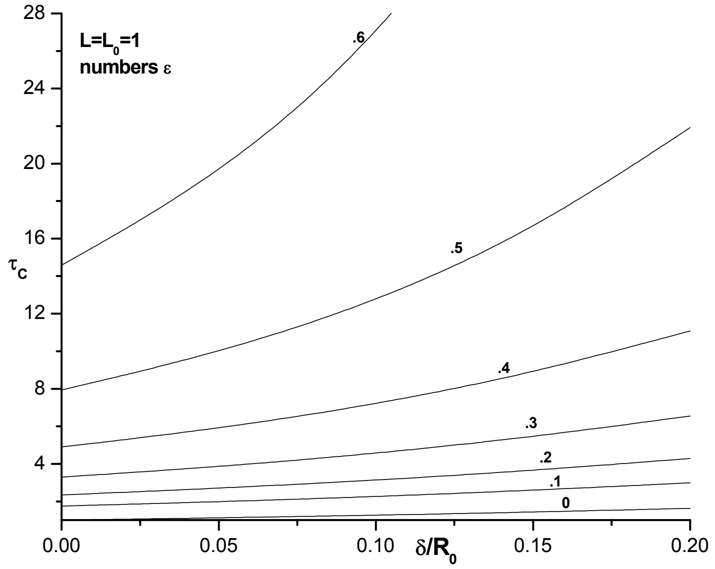


FIG. 7 Shear stress at stenosis critical height, τ_s versus δ/R_0 for different ε.

The shear stress at the stenosis throats, τ_s increases with the catheter size, ε as well as with the stenosis height, δ/R_0 (Fig.5 and 6) and possesses the characteristics similar to that of the flow resistance, λ with respect to any parameter (Figs. 2, 5). At stenosis critical height, the shear stress, τ_c too increases with catheter size, ε and stenosis height, δ/R_0 (Figs, 7, 8). However, the shear stress at the stenosis critical height, τ_c attains much smaller value than its corresponding magnitude at stenosis throats (Figs. 5, 7).

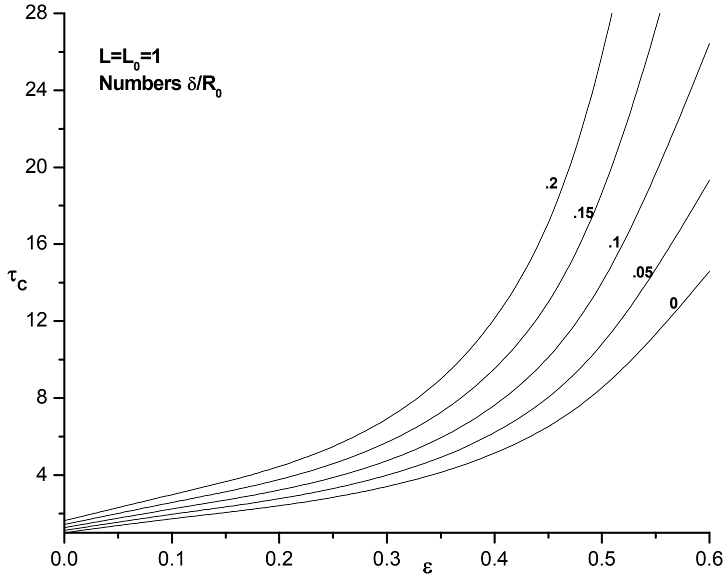


FIG. 8 Shear stress at stenosis critical height, τ_c versus ε for different δ/R_0.

Conclusions

To estimate for the increased impedance and shear stress during artery catheterization, flow through an overlapping stenosis has been analyzed assuming that the flowing blood is represented by a Newtonian fluid. The impedance increases with increasing catheter size and depends strongly on the stenosis height is an important information. Thus, the size of the catheter must be chosen keeping in view of the stenosis height during the medical treatment. The shear stress at the stenosis throats and at the stenosis critical height possess the characteristics similar to the of the flow resistance with respect to any parameter. However, the magnitude of the stenosis critical height is much smaller than its corresponding value at the stenosis throats.

The significance of the present analysis is clearly understood from the above discussion and the conclusion. The condition: $\delta/R_0 \ll 1$, limits the usefulness of the study to very early stages of the vessel constriction, which enables one the use of fully developed flow equations and leads to the locally Poiseuille like flow and closed form solutions. Parameter δ/R_0 is restricted up to 0.15 (i.e., 28% stenosis by area reduction) as beyond this value a separation in the flow may occur even at a relatively small Reynolds number (Young, 1968). The consideration of a pulsatile flow and the cases of a severe stenosis, however, are the future scope of the study. Further careful investigations are thus suggested to address the problem more realistically and to overcome the restrictions imposed on the present work.

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