Mechanics of slow draining of large cylindrical tank under gravity

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Abstract

The results of experiments on efflux time during slow draining of Newtonian liquid under gravity from a large cylindrical open storage tank through an exit pipe are presented in this paper. The mathematical equation developed by one of the authors for efflux time for single exit pipe is used to verify the experimental values. The variables studied are dia. of storage tank, dia. of exit pipe, initial height of liquid in the tank. It has been observed that during draining, Froude number remains constant and is independent of initial height of liquid in the tank. Froude number is decreased with increased exit pipe length and is also found to be independent of dia. of storage tank.

Keywords: Efflux time, cylindrical tank, exit pipe, Froude number

1. Introduction

Different geometries of storage tanks are being used by process and chemical industries. Selection of a particular geometry of tank is based on insulation requirements, floor space, corrosion requirements, material costs etc. The time required to empty the liquid content from the storage vessel is known as efflux time [1] and this is important under emergency situations as well as from productivity point of view.

Joye and Barret [2] reported mathematical equations for efflux time during draining of a Newtonian liquid (below its bubble point) from a cylindrical tank through an exit pipe (for the case of turbulent flow in the exit pipe). The authors assumed constant friction factor while developing the mathematical equation. The authors used contraction coefficient value of 1.5 for comparing the experimental values with the model values. Subbarao and co-researchers [3,4,5] made the same assumption of constant friction factor and developed equation for efflux time. The authors stated that $\frac{g_m}{g}$ is proportional to $(Fr)^2$, where g_m is modified form of acceleration due to gravity and Fr is the Froude number. The simplified equation for efflux time is named as modified form of Torricelli equation. The authors used Polyacrylamide solutions of different concentrations and reported that polymer additions decrease the efflux time and hence increase the Froude number.

Santosh Kumar and other researchers [6] developed equations for efflux time for two exit pipe systems for gravity draining of a Newtonian liquid through two exit pipe system. The authors considered two exit pipes each of $4X10^{-3}$m dia.The authors also stated that during draining, Froude number remains constant.
Subbarao and other researchers [7] also performed drag reduction experiments for two exit pipe system. The extent of increase in Froude number is observed to be more for two exit pipe systems compared to single exit pipe system in the absence and presence of polymer solutions. The authors also reported the ratio of cross sectional area of tank to pipe as 1600 below which addition of polymer solutions does not bring reduction in efflux time.
Theoretical equations for efflux time for sphere and cylinder are developed by Reddy and Subbarao [8] for the case of turbulent flow in the exit pipe. For draining the same volume of the liquid, the equations so derived are compared to find out which of the tanks considered drain faster and reported faster draining time for sphere compared to cylinder..
Subbarao [9] compared efflux times between cylinder and cone and showed that the efflux time for cone is less than that of a cylinder. Hence Froude number for cone is higher than that of a cylinder.
Gopal Singh and other researchers [10] used Polyacryl amide and polythene oxide as polymers in their studies on efflux time through exit pipe for both laminar and turbulent flow in the exit pipe and reported optimum concentrations for both laminar flow and turbulent flow in the exit pipe. They stated that the reduction in efflux time and hence increase in Froude number is influenced by type of polymer used.

All the above works increased Froude number by

- Addition of polymer solutions

  or
- Changing the geometry of the vessel

  or
- Increasing the cross sectional area by providing two exit pipes.

However, cross sectional area can also be increased by providing a single exit pipe of larger diameter. Present work considers efflux time measurement by providing $4X10^{-3}$m and $6x10^{-3}$m dia single exit pipes and compares the extent of increase in Froude number when the exit pipe dia is increased. The scope of work includes

- Efflux time measurement using water as Newtonian fluid
- Verification of these experimental values with theoretical model reported in the literature.
- Calculation of Froude number for both exit pipes considered.

2.0 Materials and methods

2.1 Description of apparatus

The schematic diagram of the apparatus and the equipment are shown in Fig 1. The equipment used consisted of known diameter tank rigidly placed on a steel structure. A mild steel pipe of known diameter (d) is welded to the tank at the centre of the bottom of the tank, served as an exit pipe. A gate valve (GV) provided at the bottom most point of the exit pipe, served as control valve for draining of liquid from the tank. A transparent plastic tube (LI) provided to the tank served as level indicator during draining operation. Efflux times are measured with a stop watch of 1 sec accuracy. The lists of experiments performed are shown in table-1.

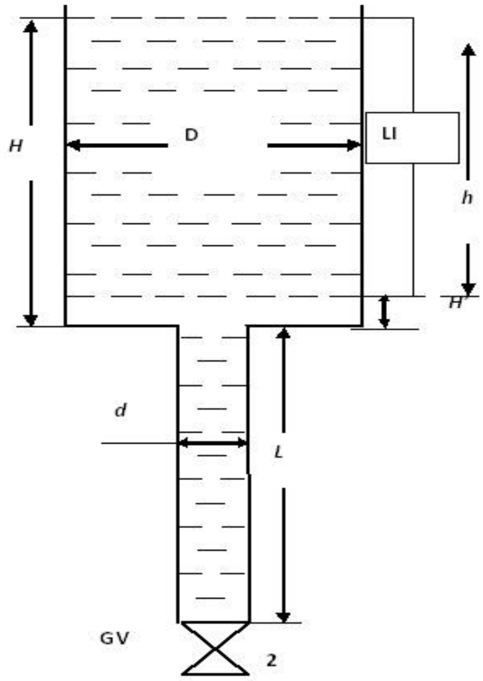

Fig. 1. Tank along with exit Pipe.

Table-1 : List of experiments

S.No	Dia. of tank, m	Length of exit pipe, m	Initial height of liquid in the tank, m
1	0.30	1,0.75,0.5,0.25	0.32. 0.26,0.20,0.14
2	0.34	1,0.75,0.5,0.25	0.38, 0.32,0.26,0.20
3	0.37	1,0.75,0.5,0.25	0.46,0.40,0.34,0.20

2.2 Experimental Procedure

Gate valve (GV) was closed and the tank was filled up to the mark and allowed to stabilize. The stopwatch was started immediately after the opening of the bottom valve. The drop in water level was read from the level indicator. The time was recorded for a fall in the liquid level to a predetermined level of above the tank bottom. The experimental efflux time is designated as ***tact***. The experiments are repeated and the measurements were taken to check the consistency of data.

3. Results and discussion

3.1 Verification of efflux time

The following efflux time equation developed by Subbarao et al (Subbarao et al, 2008) is used to verify the efflux time data.

$$t_{eff} = \sqrt{\frac{2}{g_m}}\left(\sqrt{H+L} - \sqrt{H'+L}\right) \tag{1}$$

t_{eff} is the efflux time, g_m is modified form of acceleration due to gravity given by

$$\frac{g_m}{g} = \frac{1}{\left(1 + 4f\frac{L}{d} + K_c\right)\left(\frac{A_t}{A_p}\right)^2},$$

where f is the friction factor, L length of the exit pipe, d, diameter of exit pipe, A_t and A_p are cross sectional area of tank and exit pipe respectively, K_c is the contraction coefficient whose values is reported as 1.5 by Joye and Barret [2]. This value is used for verifying the validity of the mathematical model.

The following equations for friction factor for turbulent flow reported in the literature [11, 12] are used to calculate the friction factor

$$f = \frac{0.0791}{\mathrm{Re}^{0.25}} \tag{2}$$

$$f = 0.0014 + \frac{0.125}{\mathrm{Re}^{0.32}} \tag{3}$$

To verify whether the flow is turbulent or not, Reynolds number is calculated as

$$\mathrm{Re} = DV_{2\exp}\rho/\mu \tag{4}$$

$V_{2\exp}$ is obtained using the experimentally measured data as

$$V_{2\exp} = \left[\frac{\pi}{4}D^2\left(H - H^{|}\right)/\left[\frac{\pi}{4}d^2 t_{act}\right]\right] \tag{5}$$

Where D is the dia. of tank, d is the dia. of exit pipe, H is the initial height of liquid in the tank, H' is the final height of liquid in the tank.

The density and viscosity of water as a liquid in the present study are assumed to be equal to 1000 kg/m^3 and 0.001kg/m.sec respectively. The Reynolds numbers for all the cases considered is calculated and found to be in turbulent flow only. By substituting V_{2exp}, K_c, f in eq.1 to gives t_{eff}.

The plot of $\sqrt{H+L} - \sqrt{H'+L}$ vs t_{eff} is shown in Fig.1 for a 0.3m dia. tank, 0.004m dia exit pipe and 1m exit pipe length. The efflux time obtained using eq.2 and eq.3 is represented as teq.2 and teq.3 respectively while actual efflux time is shown as tact.

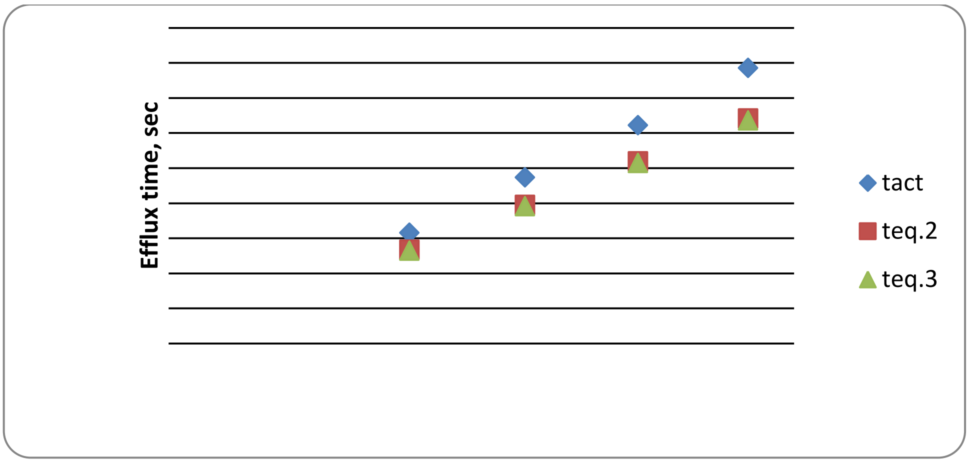

Fig.1 : Comparison of efflux time (dia.of tank=0.3m, dia.of exit pipe=0.004m and length of exit pipe=1m).

Maximum deviation of 23% is observed between experimental efflux time and teq values. However, the Eq.2 does slightly better for computing efflux time compared to Eq.3.

The trend for other exit pipe lengths and other diameters of tanks ad exit pipes is shown in the following tables (table-2 to table-7)

Table-2 : Efflux time comparison for Dia. of tank =0.30m ,exit pipe dia=4x10^{-3}m

S.No	Initial height of liquid in the tank, m	$t_{act,}$ sec	teq.2 (sec)/ teq.3 (sec)	% Error	Remarks
1	0.32	1578	1261/1250	25/26	Exit pipe length =0.75m
2	0.26	1243	1023/1015	21/23	
3	0.20	946	781/ 774	21/22	
4	0.14	632	530/525	19/20	
5	0.32	1586	1284/1274	23/24	Exit pipe length =0.5m
6	0.26	1244	1047/1040	19/20	
7	0.20	919	802/796	15/15	
8	0.14	623	548/544	14/14	
9	0.32	1598	1339/1331	19/20	Exit pipe length =0.25m
10	0.26	1249	1105/1099	13/14	
11	0.20	914	857/853	7/7	
12	0.14	620	595/592	4/5	

Table-3 : Efflux time comparison for Dia. of tank =0.34m, exit pipe dia=$4x10^{-3}$m

S.No	Initial height of liquid in the tank, m	tact, sec	teq.2,sec /teq.3,sec	% Error	Remarks
1	0.38	2045	1666/1646	23/24	Exit pipe length =1 m
2	0.32	1589	1363/1356	17/17	
3	0.26	1225	1061/1056	15/16	
4	0.20	915	758/751	21/22	
5	0.38	1985	1668/1655	19/20	Exit pipe length =0.75m
6	0.32	1650	1379/1368	20/21	
7	0.26	1280	1078/1069	19/20	
8	0.20	911	768/762	19/20	
9	0.38	2050	1688/1676	21/22	Exit pipe length =0.5m
10	0.32	1695	1401/1390	21/22	
11	0.26	1340	1103/1094	21/22	
12	0.20	960	791/785	21/22	
13	0.38	2001	1720/1712	16/17	Exit pipe length =0.25m
14	0.32	1673	1442/1434	16/17	
15	0.26	1340	1148/1142	17/17	
16	0.20	970	834/829	16/17	

Table-4: Efflux time comparison for Dia. of tank =0.37m, exit pipe dia=$4x10^{-3}$m

S.No	Initial height of liquid in the tank,m	t_{act}, sec	teq.2, sec / teq.3,sec	% Error	Remarks
1	0.46	2846	2307/2300	27/26	Exit pipe length =1 m
2	0.40	2370	1969/1943	23/22	
3	0.34	1931	1509/1487	21/21	
4	0.2	983	780/771	24/11	
5	0.46	2700	2282/2265	18/19	Exit pipe length =0.75m
6	0.40	2300	1951/1936	18/19	
7	0.34	1730	1492/1481	16/17	
8	0.2	870	770/764	13/14	
9	0.46	2700	2331	16/21	Exit pipe length =0.5m
10	0.40	2300	1949	18/15	
11	0.34	1730	1500	15/17	
12	0.2	870	783	11/11	
13	0.46	2696	2160/2149	25/25	Exit pipe length =0.25m
14	0.40	2215	1845/1836	20/21	
15	0.34	1617	1404/1397	15/16	
16	0.20	840	687/683	22/23	

Table-5 : Efflux time comparison for Dia. of tank =0.30m ,exit pipe dia=6X10^{-3}m

S.No	Initial height of liquid in the tank, m	t_{act}, sec	teq.2,sec /teq.3,sec	% Error	Remarks
1	0.32	490	442/441	11/11	Exit pipe length =1 m
2	0.26	400	358/358	12/12	
3	0.20	300	272/272	10/10	
4	0.14	190	183/183	4/4	
5	0.32	510	456/455	12/12	Exit pipe length =0.75m
6	0.26	415	371/370	12/12	
7	0.20	322	283/283	14/14	
8	0.14	228	193/193	18/18	
9	0.32	499	476/475	5/5	Exit pipe length =0.5m
10	0.26	416	390/389	7/7	
11	0.20	322	300/299	8/7	
12	0.14	227	206/205	11/10	
13	0.32	530	524/523	1/1	Exit pipe length =0.25m
14	0.26	440	434/433	12	
15	0.20	342	338/337	1/1	
16	0.14	238	235/234	1/2	

Table-6 : Efflux time comparison for Dia. of tank =0.34m, exit pipe dia = 6X10^{-3}m

S.No	Initial height of liquid in the tank, m	t_{act}, sec	teq.2, sec/teq.3	% Error	Remarks
1	0.38	615	588/587	5/5	Exit pipe length = 1 m
2	0.32	470	449/448	5/5	
3	0.26	360	342/341	5/6	
4	0.20	240	231/230	4/4	
5	0.38	610	601/600	1/2	Exit pipe length =0.75m
6	0.32	470	461460	2/2	
7	0.26	363	352/351	3/3	
8	0.20	242	239/238	1/ 2	
9	0.38	640	626/625	2/2	Exit pipe length =0.5m
10	0.32	492	483/482	2/2	
11	0.26	381	371/370	3/3	
12	0.20	260	253/253	3/3	
13	0.38	680	675/675	1/1	Exit pipe length =0.25m
14	0.32	530	528/527	0.4/0.5	
15	0.26	420	410/409	2/3	
16	0.20	290	284/283	2/2	

S.No	Initial height of liquid in the tank, m	$t_{act,}$ sec	teq.2, sec /teq.3,sec	% Error	Remarks
1	0.46	907	857/856	6/6	Exit pipe length = 1 m
2	0.40	735	697/696	5/6	
3	0.34	539	530/529	2/2	
4	0.2	361	360/359	0.4/0.6	
5	0.46	902	874.872	3/3	Exit pipe length =0.75m
6	0.40	731	713/712	3/3	
7	0.34	547	545/544	0.3/0.5	
8	0.2	376	372/371	1/1	
9	0.46	902	902/901	0	Exit pipe length =0.5m
10	0.40	742	741/740	0.16/0.16	
11	0.34	572	571/570	0.1/0.3	
12	0.2	395	393/392	0.5/1	
13	0.46	968	963/962	0.5/0.6	Exit pipe length =0.25m
14	0.40	800	800/799	0/0.2	
15	0.34	630	625/624	1/1	
16	0.20	450	437/436	3/3	

Table-7 : Efflux time comparison for Dia. of tank =0.37m , exit pipe dia=$6X10^{-3}$m

It can be seen from the tables (Table-2 to table-7) that the deviation is very less when the exit pipe dia. is $6X10^{-3}$m.

3.2 Variation of Froude number with initial height of liquid in the tank:

Froude number is defined by Subbarao and other researchers as

$$\frac{g_m}{g} = \frac{\left(1 + 4f\frac{L}{d} + K_c\right)}{\left(\frac{A_t}{A_p}\right)^2} \quad \text{where } \frac{g_m}{g} \text{ is proportional to } (Fr)^2 \ [3]$$

The following plot shows variation of Froude number with initial height of liquid in the tank for 0.30m dia. tank while keeping the length of the exit pipe at 1m.

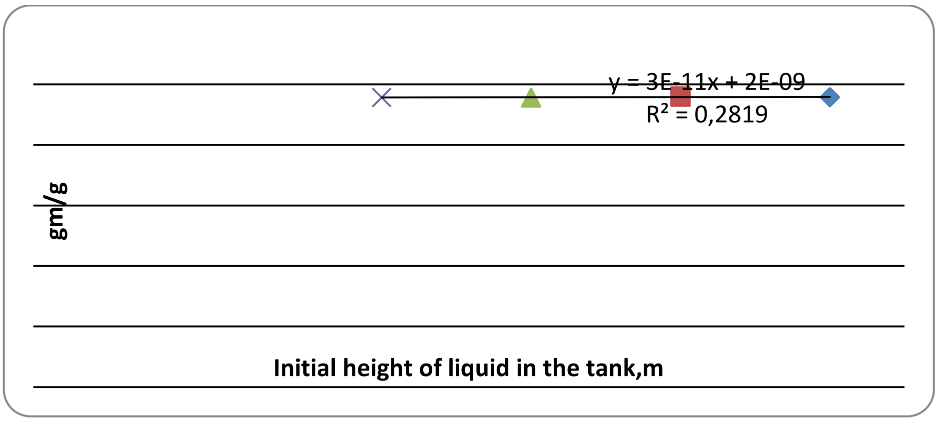

Fig.2 : Variation of g_m/g with initial height of liquid in the tank.

The plot suggests that Froude number is independent of initial height of liquid in the tank. However, when the length of exit pipe is increased, variation of Froude number with length of exit pipe is shown below.

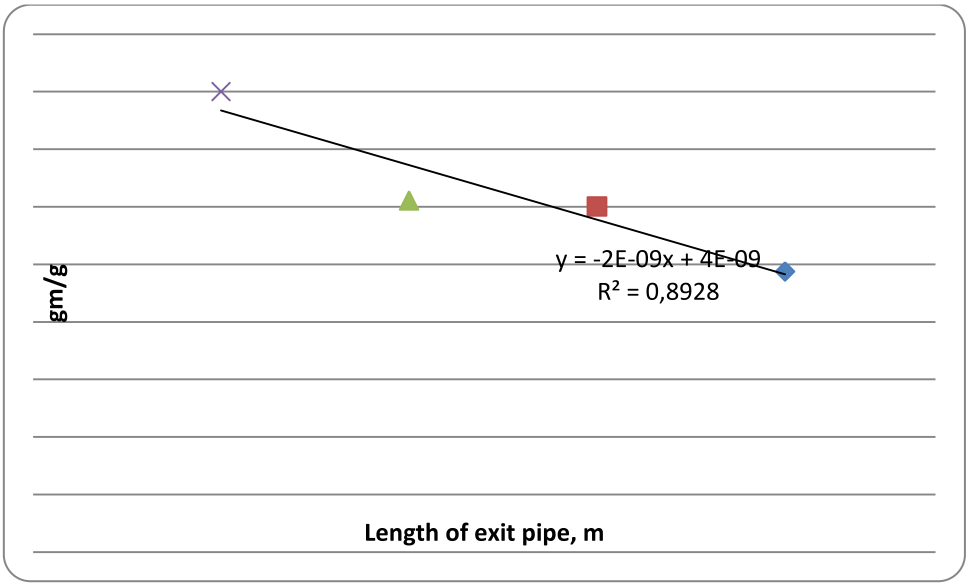

Fig.3 : Variation of efflux time with length of exit pipe

It can be seen from the plot that as the length of exit pipe increases, Froude number decreases**.**

The variation of $\frac{g_m}{g}$ for different dia. of tanks and exit pipes are shown in the following tables (tables-8 and table-9).

Table- 8 : Variation of $\frac{g_m}{g}$ for different dia.of tanks.

S.No	Initial height of liquid in the tank,m	g_m/g X10^9	Length of exit pipe, m	Remarks
1	0.32,0.26,0.20,0.14	2.4	1	Dia. of storage tank =0.30m and Dia. of exit pipe =4X10-3,m
2	0.32,0.26,0.20,0.14	3	0.75	
3	0.32,0.26,0.20,0.14	4	0.5	
4	0.32,0.26,0.20,0.14	6	0.25	
5	0.38,0.32,0.26,0.2	2.4	1	Dia. of storage tank =0.34m and Dia. of exit pipe =4X10-3m
6	0.38,0.32,0.26,0.2	3	0.75	
7	0.38,0.32,0.26,0.2	4	0.5	
8	0.38,0.32,0.26,0.2	6	0.25	
9	0.46,0.40,0.34,0.2	2.4	1	Dia. of storage tank =0.37m and Dia. of exit pipe =4X10^{-3}m
10	0.46,0.40,0.34,0.2	3	0.75	
11	0.46,0.40,0.34,0.2	4	0.5	
12	0.46,0.40,0.34,0.2	6	0.25	

Table- 9 : Variation of $\frac{g_m}{g}$ for different dia.of tanks.

S.No	Initial height of liquid in the tank,m	g_m/g X10^8	Length of exit pipe, m	Remarks
1	0.32,0.26,0.20,0.14	2	1	Dia. of storage tank =0.30m and Dia. of exit pipe =6X10^{-3},m
2	0.32,0.26,0.20,0.14	2.4	0.75	
3	0.32,0.26,0.20,0.14	3	0.5	
4	0.32,0.26,0.20,0.14	4	0.25	
5	0.38,0.32,0.26,0.2	2	1	Dia. of storage tank =0.34m and Dia. of exit pipe =6X10^{-3}m
6	0.38,0.32,0.26,0.2	2.4	0.75	
7	0.38,0.32,0.26,0.2	3	0.5	
8	0.38,0.32,0.26,0.2	4	0.25	
9	0.46,0.40,0.34,0.2	2	1	Dia. of storage tank =0.37m and Dia. of exit pipe =6X10^{-3}m
10	0.46,0.40,0.34,0.2	2.4	0.75	
11	0.46,0.40,0.34,0.2	3	0.5	
12	0.46,0.40,0.34,0.2	4	0.25	

It can be seen from the tables (Tables- 8 and 9) that Froude number is independent of dia. of storage tank and depends only on the dia and length of the exit pipe. It can also be seen that as the dia. of exit pipe increases, Froude number also increases.

4. **Conclusions**

Some of the conclusions of the above study are

a. The difference in experimental values and theoretical values of efflux time using a contraction coefficient of 1.5 leads to a maximum deviation of 25% for 4X10-3m dia exit pipe where is the maximum deviation is 11% in case of 6X10-3m dia exit pipe.
b. Froude number remains constant and is independent of initial height of liquid in the tank. It is also indepedent of dia. of storage vessel when drained by exit pipe of same dia.
c. Froude number increases when the length of exit pipe is decreased.
d. Froude number also increased with increased dia. of exit pipe.

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