A Two-Layered Blood Flow Through a Narrow Catheterized Artery

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Absract

The modeling of two-layered blood flow an inserted catheterized artery has been invstigated. The flowing blood is considered to be incompressible, Newtonian with variable blood viscosity. The functional dependence of blood viscosity on haematcroit (percentage of red cells) has been duly accounted for in order to improve resemblance to the real solution. It is noticed that the effective viscosity, flow rate and arterial wall shear stress in particular, in the catheterized site, are significantly altered. Flow characteristics assumes, its lower magnitude in catheterized artery as compared to the uncathetrized artery for any given set of parameter. Numerical results reveal that the effective viscosity in minimal magnitude and consequently the flowrate assumes its maximal magnitude during the artery catheterization at the catheter size approximately fifty percent to that of artery size.

Introduction

The study of blood flow through an inserted catheter has been the subject of scientific research for a long time. Its plays an important role in the fundamental understanding, diagnosis and treatment of cardiovascular system. Like most of the problem of nature and life sciences. It is complex one due to the complicated structure of blood, the circulatory system and their constituent materials. The experimental studies and the theoretical treatment of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in human or animal physiology and for other clinical purposes and practical applications. Mathematical Modelling of blood flow has been subject to modification in order to account for the new evidence uncovered through the improved initial experimental observation (Srivastava and Srivastava, 1983).
Blood is composed of fluid plasma and formed elements. The formed elements of blood are erythrocyte, leukocyte and platelets. The percentage volume of red cells is called the haematocrit and is approximately 40- 45 % (Oka,1981) for an adult. Red cells may effect the viscosity of whole blood and thus the velocity distribution dependa on concentration of cells. So blood can not considered as homogenous fluid. In general, Blood is known to be an incompressible non-Newtonian Fluid. This property is mainly the result of cell concentration (haemotocrit ratio) (H. Demiray,2008). However in the course of flow in arteries, the red blood cells in the vicnity of arterial wall move to the central region of the artery.So that the haematocrit ratio becomes quite low near the arterial wall, which results in lower viscosity in this region. Moreover, due to high shear rate near the arterial wall, the viscosity of blood id further reduced. Hence blood may be treated as a Newtonian fluid with variable viscosity particularly in case of flow through larger vessels.
In recent times, with the evaluation of coronary balloon angioplasty, there has been a considerable increase in the use of catheter of various size. The insertion of a catheter in

an artery will alter the flow field, modify the pressure distribution and increase the resistance. Therefore the pressure gradient recorded by a transducer attached to the catheter will differ from that of uncatheterised artery and It is essential to know that catheter induced error. The use of catheter is of immense importance and has become standard tool for diagnosis and treatment in modern medicine. A catheter is made of polyester based thermoplastic polyurethane, medical grade polyvinyl chloride etc. For the purpose of flexibility PVC materials containing added plasticizers are used in catheter which enable them to move through the branches or curved paths of the circulatory system. Transducers attached to catheters are of large usage in clinical works and the techniques is used for measuring blood pressure or other mechanical properties in arteries [Garbe, 1972; Anderson etal. 1986]

Supported by the experimental results on controlled dog, Kanai etal. (1970) established theoretically that to reduce the error at the tip of the catheter for each experiment, a catheter of appropriate size (diameter) is needed, Bojorno etal. (1976,1977a, 1977b) and Hellsten and Petterson (1977) experimentally studied the hydro and homodynamic effects of catheterijation of vessels in a series of papers. Back and Denton (1992) obtained the estimate at the wall shear stress and discussed its clinical importance in coronary angioplasty. Back (1994) and Back etal. (1996) studied the important hemodynamic characterstic like the wall shear stress, pressure drop and frictional region in catheterized coronary artery under normal as well as the pathological situation of stenosis present.

The effect of catheterization on various flow characterstic in a curved artery was studied by Karahalios (1990) and Jayaraman and Tiwari(1995). In all the investigation blood has treated as a newtonian fluid. Dash etal. (1996) studied the changed flow pattern in narrow artery when a catheter is inserted into it and estimate the increase in friction in the artery due to catheterization using casson fluid model for steady and pulsatile flow of blood. Recently, Sankar and Hemlata (2007) addressed the problem of pulsatile flow of Hershel-Bulkey fluid in catheterized arteries. Numerous investigations have cited hydrodynamic factors playing an important role in an insertion catheter and mathematical modeling through insertion catheter is also very important.

In this paper, an attempt has been made to study the effect of catheterization on physiologically important flow quantities such as effective viscosity, flow rate and wall shear stress for a two layered blood flow in a narrow artery by modeling blood as newtonian homogenous incompressible fluid. The model consists of a core surrounded by a peripheral layer.It is assumed that the fluid of both the region (core and peripheral) are Newtonian having different viscosities. In the model, the flow of blood is represented bu double layered model. Bugliarello and Sevilla (1970) and Hayden (1963) have experimentally observed that when blood flows through narrow tubes there exists a cell free plasma layer near the wall. In view of their experiments, It is preferable to represent the flow of blood through narrow tubes by a two layered model instead of single layered model.

Formulation of the Problem

Consider a two layered model for the blood flow with in a circular cylinder tube of radius, **a** with an inserted catheter of radius, $\mathbf{a_c}$, It is assumed that blood is represented by a two layered model with in a central layer (core region) of a homogenous and incompressible fluid, and a pheripheral layer of plasma (Newtonian fluid) of thickness, $\mathbf{a_1}$ as shown in Fig.1.

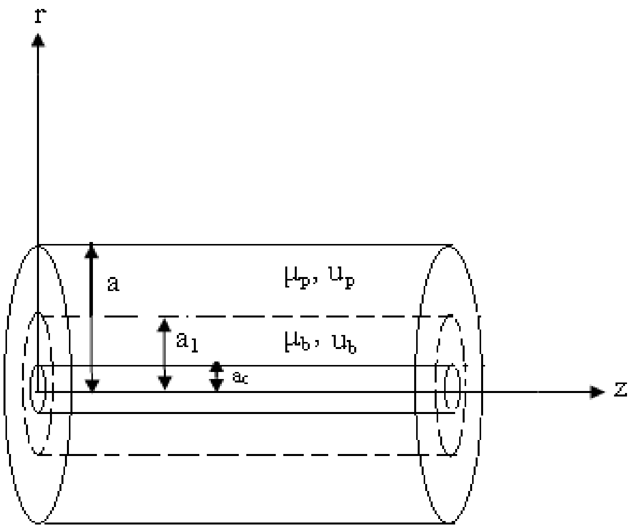

Fig.1 Flow geometry in an artery with an inserted catheter.

The equation governing the conservation of mass and linear momentum for the steady flow of blood are expressed as

$$\frac{dp}{dz} = \frac{\mu_p}{r} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} \right) u_p \tag{1}$$

$$\frac{dp}{dz} = \frac{\mu_b}{r} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} \right) u_b \tag{2}$$

Where r is the radial coordinate, u_p and u_b, respectively the velocity in the peripheral and core layer in the axial direction. P is hydraulic pressure and (μ_p, μ_b) are the viscosity of blood in the pheripheral and core layer region, respectively.
Blood is viscoelastic fluid and its rheological properties, viscosity and elasticity depend on the rate of flow and shear stress.The blood flow in the artery is due to a constant pressure gradient along the artery axis. The artery length is assumed to be large enough as compared to its radius so that the entrance end the special wall effect can be neglected.
In addition, the whole blood is a Newtonian fluid. An attempt to analyze the system in an exact manner is very difficult due to the complicated structure of blood and the circulary system.

The flow is subject to the following boundary condition.

(i) No slip condition are assumed at the artery and catheter wall.

$$u_p = 0 \qquad \text{at } r = a \text{ (artey wall)}$$

and $u_b = 0$ at $r = a_c$ (catheter wall) (3)

(ii) The velocity and shear stress are continuous at interface of peripheral and central layers

$$u_p = u_b \quad \text{and} \quad \mu_p \frac{\partial u_p}{\partial r} = \mu_b \frac{\partial u_b}{\partial r} \qquad \text{at } r=a_1 \tag{4}$$

Analysis

The solution of equation (1) and (2) subject to the boundary condition (3) and (4) is given by.

$$u_p = -\frac{a^2}{4\mu_p}\frac{dp}{dz}\left\{1-\left(\frac{r}{a}\right)^2 + N\log\frac{r}{a}\right\} \tag{5}$$

$$u_b = -\frac{a^2}{4\mu_b}\frac{dp}{dz}\left\{\varepsilon^2-\left(\frac{r}{a}\right)^2 + N\log\frac{r}{a_c}\right\} \tag{6}$$

Where $N = M/a^2 = \dfrac{(a_1/a)^2 - 1 - \mu\{(a_1/a)^2 - \varepsilon^2\}}{\log(a_1/a) - \mu\log(a_1/a_c)}$

and non-dimensional $\varepsilon = a_c/a$, $\delta = a_1/a$ $\mu = \mu_p/\mu_b$

The flow rate $Q = Q_p + Q_c$

where

$$Q_p = 2\pi\int_{a_1}^{a} r\,u_p\,dr \qquad \text{and} \qquad Q_c = 2\pi\int_{a_c}^{a_1} r\,u_b\,dr \tag{7}$$

Q_p and Q_c are the flux through peripheral layers and central layers, respectively.

The flow rate

$$Q = -\frac{\pi a^4}{8\mu_p}\frac{dp}{dz}\left[1+(1-\mu)\alpha^4 - 2\alpha^2(1-\mu\varepsilon^2) - \mu\varepsilon^4 - N\{1-(1-\mu)\alpha^2 - \mu\varepsilon^2 + 2\alpha^2(\log\alpha - \mu\log(\alpha/\varepsilon))\}\right] \tag{8}$$

The expression for the effective viscosity, μ_{eff} may now be written from

$$\mu_{eff} = \frac{\mu_p}{1+(1-\mu)\alpha^4 - 2\alpha^2(1-\mu\varepsilon^2) - \mu\varepsilon^4 - N\{1-(1-\mu)\alpha^2 - \mu\varepsilon^2 + 2\alpha^2(\log\alpha - \mu\log(\alpha/\varepsilon))\}} \tag{9}$$

The pressure drop Δp in a tube of length, L is calculated as

$$\Delta p = -\int_0^L \left(\frac{dp}{dz}\right)dz$$

$$= \frac{8QL}{\pi a^4}\left[\frac{\mu_p}{1+(1-\mu)\alpha^4-2\alpha^2(1-\mu\varepsilon^2)-\mu\varepsilon^4-N\{1-(1-\mu)\alpha^2-\mu\varepsilon^2+2\alpha^2(\log\alpha-\mu\log(\alpha/\varepsilon))\}}\right] \quad (10)$$

The non-dimensional wall shear stress (frictional resistance or resistive impedance) at the artery wall, τ_a is now calculated as

$$\tau_a = \lambda'/\lambda_p = \mu_{eff}/\mu_p = \frac{1}{1+(1-\mu)\alpha^4-2\alpha^2(1-\mu\varepsilon^2)-\mu\varepsilon^4-N\{1-(1-\mu)\alpha^2-\mu\varepsilon^2+2\alpha^2(\log\alpha-\mu\log(\alpha/\varepsilon))\}} \quad (11)$$

Where $\lambda' = \Delta p / Q$ and $\lambda_p = \frac{8\mu_p L}{\pi a^4}$, λ_p being the flow resistance for a Newtonian Fluid of viscosity, μ_p (peripheral fluid) in the absence of the catheter.

The dimensionless shear stress at the catheter wall is thus obtained as

$$\tau_c = \frac{\tau_c'}{\tau_p} = \frac{\varepsilon}{1+(1-\mu)\alpha^4-2\alpha^2(1-\mu\varepsilon^2)-\mu\varepsilon^4-N\{1-(1-\mu)\alpha^2-\mu\varepsilon^2+2\alpha^2(\log\alpha-\mu\log(\alpha/\varepsilon))\}} \quad (12)$$

with $\tau_c' = (-a_c/2)\frac{dp}{dz}$ and $\tau_p = \frac{4\mu_p Q}{\pi a^3}$, τ_p being the shear stress for a Newtonian fluid of viscosity μ_p (peripheral fluid) in the absence of the catheter.

When μ=1 ($\mu_p = \mu_b$) and δ=1 (a_1=a i.e. in the absence of peripheral layer) the result of present analysis reduce to single layer model with inserted catheter for C=0 Srivastava et al.[2009].

$$Q = -\frac{\pi a^4}{8\mu_p}\frac{dp}{dz}\{1-\varepsilon^4+(1-\varepsilon^2)^2/\log\varepsilon\}, \quad (13)$$

$$\mu_{eff} = \frac{\mu_p}{1-\varepsilon^4+(1-\varepsilon^2)^2/\log\varepsilon}, \quad (14)$$

$$\tau_a = \frac{1}{1-\varepsilon^4+(1-\varepsilon^2)^2/\log\varepsilon}, \quad (15)$$

$$\tau_c = \frac{\varepsilon}{1-\varepsilon^4+(1-\varepsilon^2)^2/\log\varepsilon} \quad (16)$$

In the absence of the catheter (i.e. under the limit $\varepsilon \to 0$) Inner tube and taken δ=1, the peripheral layer disappear and viscosity $\mu_p = \mu_b$, one now derives the expression for corresponding flow charactersties in a classical Poiseuille flow from equation [8-12]

Result and Discussion

In order to discuss the result of the theoretical model proposed in the study quqntitavely and to point out its biological relevance. Computer codes are developed to evaluate the analytical results for effective viscosity, flow rate and shear stress obtained in equation (7)- equation (9) for various value of the parameter involved. Due to non-availbility of any such theoretical or experimental studies of double layer blood flow through an artery with an inserted catheter, the result of the present study shall be compard with corresponding results obtained in the theoretical model of Srivastava et al. [1994] in absence of external body acceleration and also with the classical Poisuille fllow of single-phase Newtonian viscous fluid. The result of the present study under δ=1 (peripheral layer disappears) and (μ=1),correspond to those obtained from the single layer two phase macroscopic model for blood flow used in Srivastava et al. [2009] for C=0 in the absence of external body acceleration. For μ=1 and δ=1, the result of study reduce to the case of the flow of Newtonian Fluid through a catheterized artery. Under the limit ε →0, μ=1and δ=1, the present study reduce to the case of a classical Poiseuille flow.

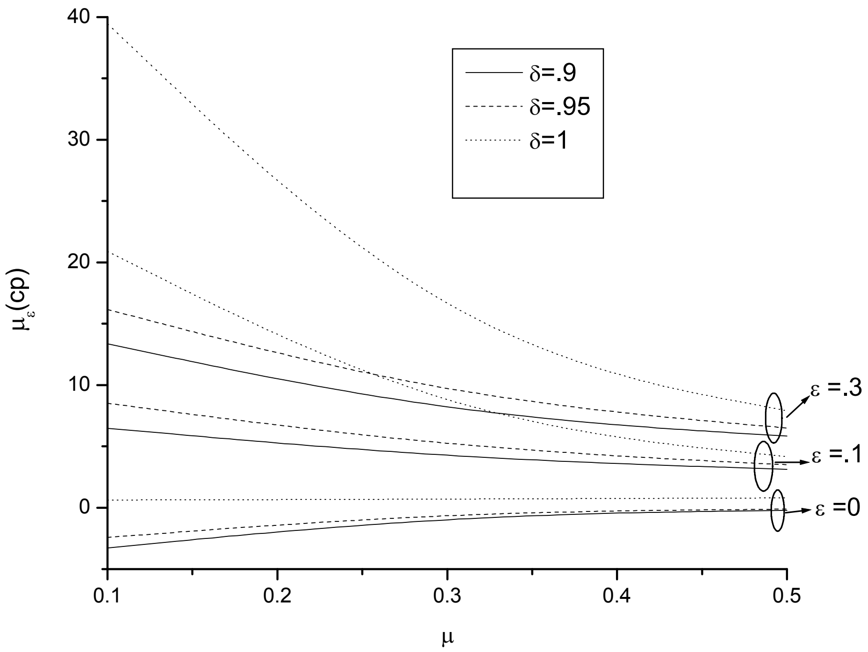

Figure.2 Effective viscosity, μ_{eff} versus viscosity, μ for differnt thickness of peripheral layer, δ and catheter size, ε in a 70 μm diameter artery.

One observe that the effective viscosity, μ_{eff} decreases with viscosity, μ for any given pheripheral thickness, δ and the catheter size, ε(>0) and for the ε =0 the effective viscosity slightly increases (Fig.2). For any given set of other parameter, μ_{eff} is found to be increasing for small catheter size, ε $(0 \le \varepsilon \le 0.1)$ but the properties deviates for large catheter size, ε (Fig.3). Numericals results indicate that the increase in the $\mu_{eff,}$ assumes its maximum magnitude at ε=.5 beyond this value of ε, the flow characterstics, μ_{eff} increases without bound. However the effective viscosity, μ_{eff} obtain in a classical Poiseuille flow (μ=1 & δ=1) are 1.2 for ε=0 (without catheter) and 1.756185, 1.697289 and 1.584863 (with catheter), is independent of artery size.

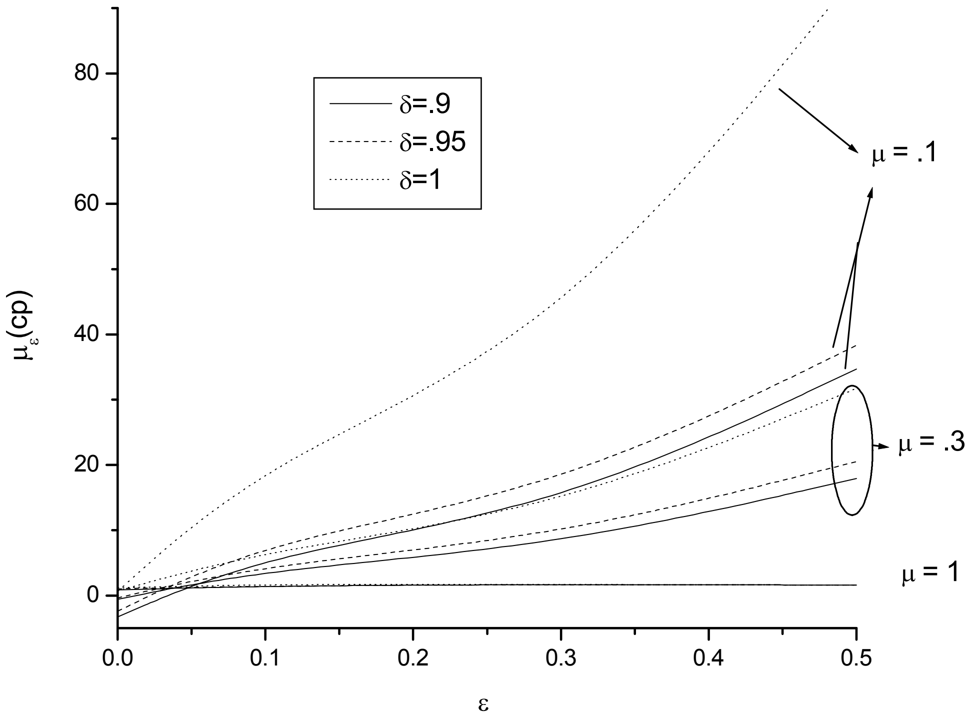

Figure.3 Effective viscosity, μ_{eff} versus catheter size, ε for differnt thickness of peripheral layer, δ and viscosity, μ in a 70 μm diameter artery.

The volumetric flowrate, Q increases with the pressure gradient, -dp/dz, for catheter size ε =.1, and for a given value of pheripheral thickness δ, and viscosity of blood, μ and observe that on increasing the value of any given set of parameter, μ, the flow rate decrease at the catheter size ε =.1(Fig. 4) and flowrate in the uncathetrized artery (ε=0) at pressure 76mm lower in magnitude and on coming the presence of catheter its rises.However, in the case of catherized artery its magnitude is found to be ~ maximum for the value of ε=0.5, show similar nature of variation in Q as the flow characterstic μ_{eff} and numerical result for δ=1 (pheripherel layer disappears) and (μ=1) similar to the result Srivastava and Srivastava [2009] for C=0 single layer model (Fig.5).

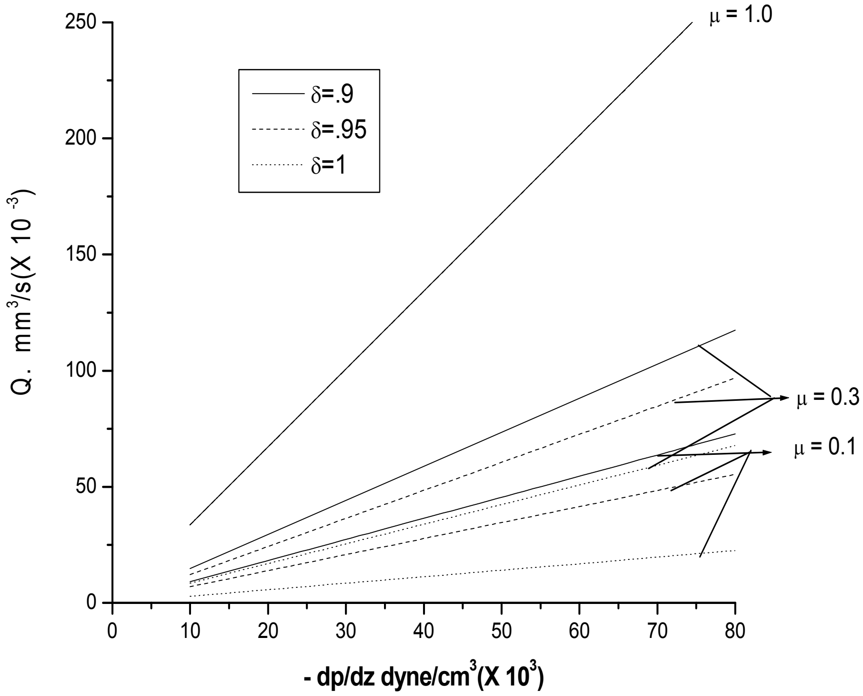

Figure.4 Pressure-flow rate relationship for different viscosity, μ and thickness of peripheral layer, δ at the catheter thickness ε=.1 in a 70 μm diameter.

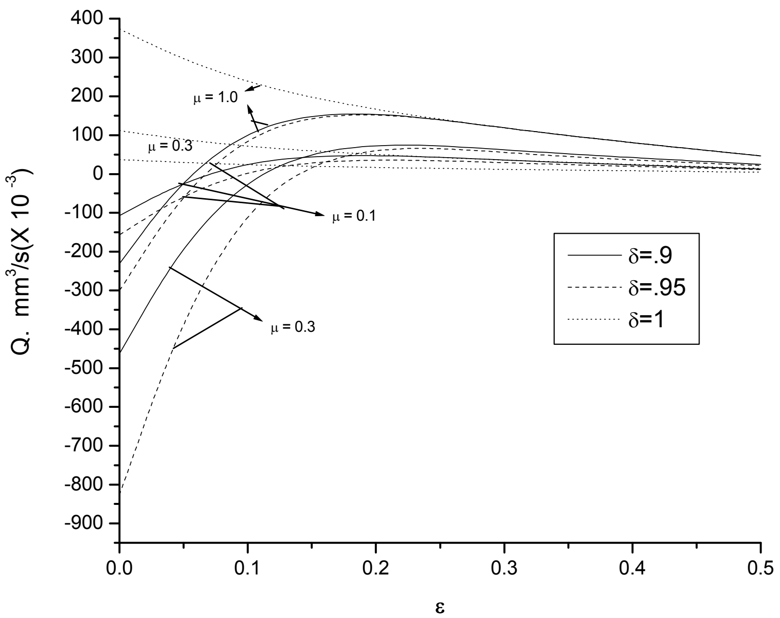

Figure.5 Flow-rate versus catheter size, ε for different viscosity, μ and thickness of pheripheral layer, δ in a 70 μm diameter artery.

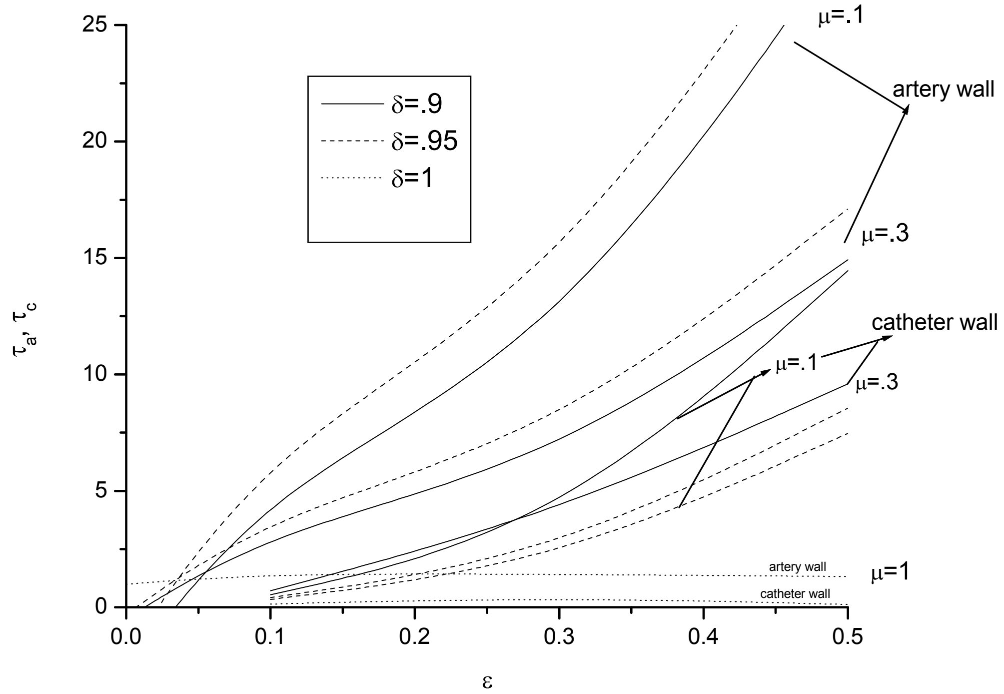

Figure. 6 Non-dimensional shear stress τ_a and τ_c versus catheter size, ε for different thickness of pheripheral layer,δ in a 70 μm diameter artery.

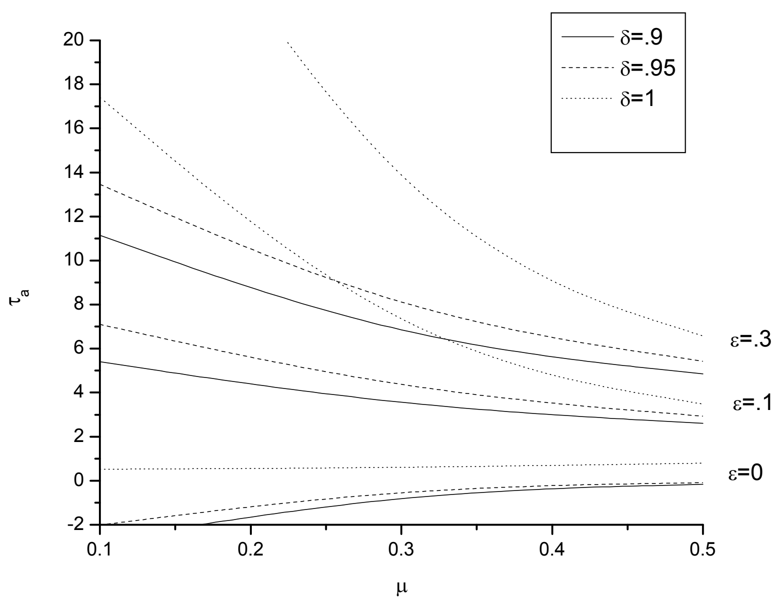

Figure.7 Non-dimensional shear stress τ_a, versus viscosity, μ for different thickness of pheripheral layer, δ in a 70 μ m diameter artery.

It is display from Fig. 6 that shear stress at the artery wall τ_a and catheter wall, τ_c increase in the presence of catheter and similar result as the flow characterstic μ_{eff} with respect to parameter, ε and shear stress is greater than at the artery wall compare to the catheter wall and for δ=1 (pheripherel layer disappears) and (μ=1) similar to the result Srivastava [2009] for C=0 single layer model.

The non- dimensional shear stress at artery wall, τ_a and catheter wall, τ_c in Fig. 7 and 8, decrease viscosity, μ for any catheter size, ε. It is noticed that shear stress at artery wall, τ_a always assumes higher magnitude than the corresponding magnitude of the shear stress at catheter wall, τ_c and on increasing the catheter size shear stress τ_a and τ_c increase and when δ=1 (pheripheral layer disappear) than shear stress higher magnitude as compare to the presence of pheripheral layer.

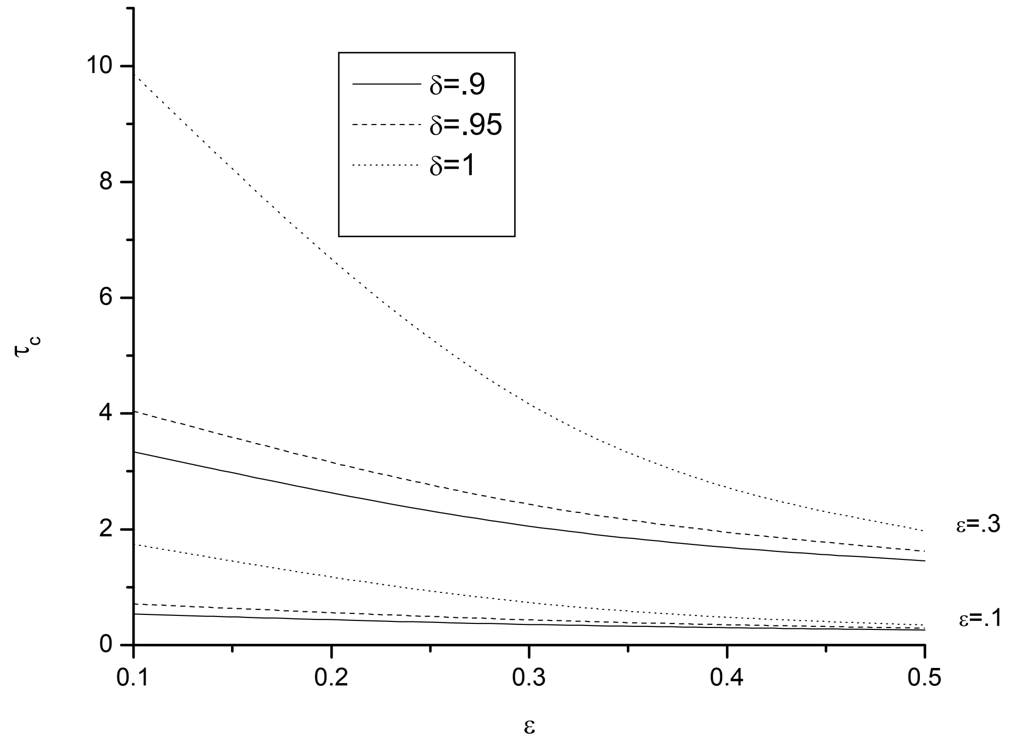

Figure.8 Non-dimensional shear stress τ_c, versus viscosity, μ for different thickness of pheripheral layer,δ in a 70 μ m diameter artery.

Conclusion

A two-layered model of blood flow through a catheterized artery with axially variable peripheral layer thickness and variable viscosity at the wall has been considered. The effective viscosity and other flow characterstics are highly influenced by the presence of catheter.Numerical results reveal that the effective viscosity in minimal magnitude and consequently the flowrate assumes its maximal magnitude during the artery catheterization at the catheter size approximately fifty percent to that of artery size.The understanding gained through this study may contribute to the development of more generalized model for the suspension double layered blood flow model and its role in the red cell concentration.

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