BLOOD FLOW THROUGH A COMPOSITE STENOSIS IN CATHETERIZED ARTERIES

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Abstract- The Problem of blood flow through a composite stenosis during artery catheterization assuming blood to behave like a Newtonian fluid, has been investigated. The analytical expression for the blood flow characteristics, namely, the impedance, the wall shear stress in the stenotic region and the shear stress at stenosis throat has been derived. The impedance increases with the catheter as well as with the stenosis size (height and length both). The wall shear stress in the stenotic region increases in the up stream of the stenosis throat but decreases in the down stream of the throat and achieves to its approached value at a critical axial distance. The magnitude of the wall shear stress at the end point of the constriction profile is much lower than its approached value. The shear stress at the stenosis throat possesses the characteristics similar to that of the flow resistance with respect to any given parameter.

Key words: Impedance, catheter, shear stress, throat, stenotic region

INTRODUCTION

The generic medical term ‘Stenosis’ means a narrowing of any body passage, tube or orifice, and is a frequently occurring cardiovascular disease in mammalian arteries. Stenosis or arteriosclerosis is the abnormal and unnatural growth in the arterial wall thickness that develops at various locations of the cardiovascular system under diseased conditions which occasionally results into serious consequences (myocardial infraction, cerebral strokes, argina pitoris, etc.). It is believed that stenoses are caused by the impingement of extravascular masses or due to intravascular atherosclerotic plaques which develop at the wall of the artery and protrude into the lumen. Regardless of the cause, it is well established that once an obstruction has developed, it results into significant changes in blood flow, pressure distribution, wall shear stress and the impedance (flow resistance). In the region of narrowing arterial constriction, the flow accelerates and consequently the velocity gradient near the wall region is steeper due to the increased core velocity resulting in relatively large shear stress on the wall even for a mild stenosis.

The use of catheters is of immense importance and has become standard tool for diagnosis and treatment in modern medicine. Transducers attached to catheters are of large usage in clinical works and the technique is used for measuring blood pressure and other mechanical properties in arteries. When a catheter is inserted into the stenosed artery the further increased impedance or frictional resistance to flow will alter velocity distribution. Recently, Srivastava and Srivastava (2009) presented a review of most of the experimental and theoretical investigations on artery catheterization. Kanai et al. (1970) established analytically that for each experiment, a catheter of appropriate size is required in order to reduce the error due to the wave reflection at the tip of the catheter. To treat arteriosclerosis in balloon angioplasty, a catheter with a tiny balloon attached at the end is inserted into the artery. The catheter is carefully guided to the location at which stenosis occurs and the balloon is inflated to fracture the fatty deposits and widen the narrowed portion of the artery. Gunj et al. (1985), Anderson et al. (1986) and Wilson et al. (1988) have studied the measurement of translational pressure gradient during angioplasty. Leimgraber et al. (1985) have reported high mean pressure gradient across the stenosis during balloon angioplasty. The mathematical model of artery catheterization corresponds to the flow through an annulus. The mean flow resistance increase during catheterization in normal as well as in stenosed arteries has been studied by Back (1994) and Back et al. (1996). Sarkar and Jayaraman (1998) discussed the changed flow patterns of pulsatile blood flow in a catheterized stenosed artery. Dash et al. (1999) further studied the problem in a stenosed curved artery. Sankar and Hemlatha (2007) studied the flow of Herschel-Bulkley fluid in a catheterized blood vessel. Most recently, Srivastava and Rastogi (2009, 2010) have discussed the macroscopic two-phase flow of blood in stenosed catheterized arteries.

A survey of the literature on arteriosclerotic development indicates that the studies conducted in the literature are mainly concerned with the single symmetric and non-symmetric stenoses. The stenoses may develop in series (multiple stenoses) or may be of irregular shapes or overlapping or of composite in nature. An effort is made in the present work to estimate the increased impedance and other flow characteristics during artery catheterization in an artery with a composite stenosis assuming that the flowing blood to behave like a Newtonian fluid. The wall in the vicinity of the stenosis is usually relatively solid when stenoses develop in the living vasculature. The artery length is considered large enough as compared to its radius so that the entrance, end and special wall effects can be neglected.

FORMULATION OF THE PROBLEM

Consider the axisymmetric flow of blood through a catheterized artery with a composite stenosis. The geometry of the stenosis which is assumed to be manifested in the arterial segment is described in Fig. 1 as

$$R(z)/R_0 = 1 - \frac{2\delta}{R_0 L_0}(z-d); \qquad d \le z \le d + L_0/2, \tag{1}$$

$$= 1 - \frac{\delta}{2R_0}\left\{1 + \cos\frac{2\pi}{L_0}(z - d - L_0/2)\right\};$$

$$d + L_0/2 \le z \le d + L_0, \tag{2}$$

$$=1;$$

$$\text{otherwise,} \tag{3}$$

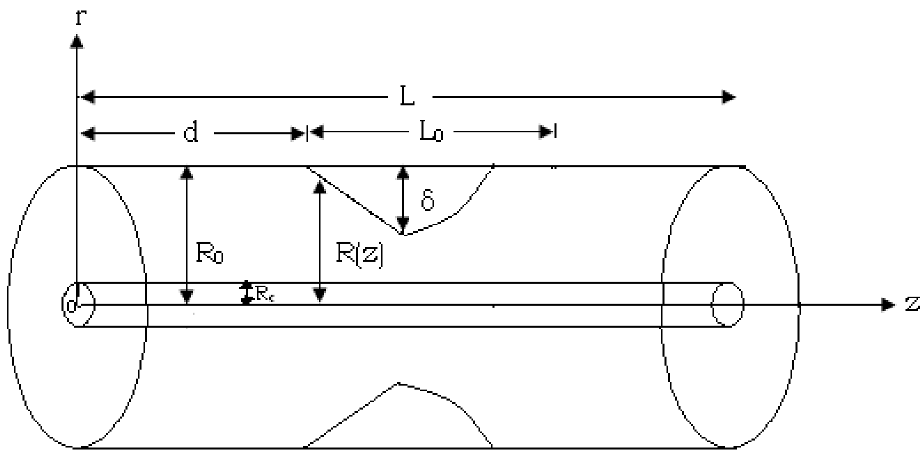

Fig. 1 Geometry of a composite stenosis in a catheterized artery.

where $R \approx R(z)$ and R_0 are the radius of the artery with and without stenosis, respectively, R_c is the radius of the catheter. L_0 is the length of the stenosis and d indicates its location, δ is the maximum projection of the stenosis at $z = d + L_0/2$.

Blood is assumed to be represented by a Newtonian fluid and following the report of Young (1968) and considering the axisymmetric, laminar, steady, one-dimensional flow of blood in an artery, the general constitutive equation in a mild stenosis case, under the conditions (Young, 1968; Srivastava and Rastogi, 2009): $\delta/R_0 << 1$, $R_e(2\delta/L_0) << 1$ and $2R_0/L_0 \sim O(1)$, may be stated as

$$\frac{dp}{dz} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u, \tag{4}$$

where (r,z) are cylindrical polar coordinates system with z measured along the tube axis and r measured normal to the axis of the tube, R_e is the tube Reynolds number, p is the pressure and (u, μ) is the fluid (velocity, viscosity).

The boundary conditions are

$$u = 0 \quad \text{at} \quad r = R(z), \tag{5}$$

$$u = 0 \quad \text{at} \quad r = R_c, \tag{6}$$

Conditions (5) and (6) are the standard no slip conditions at the artery and catheter walls, respectively.

ANALYSIS

The expression for the velocity obtained as the solution of equation (4) subject to the boundary conditions (5) and (6), is given as

$$u = -\frac{R_0^2}{4\mu}\frac{dp}{dz}\left[(R/R_0)^2 - (r/R_0)^2 + \frac{\left\{(R/R_0)^2 - (R_c/R_0)^2\right\}}{\log(R/R_c)}\log\left(\frac{r}{R}\right)\right]. \quad (7)$$

The flow flux, Q is thus calculated as

$$Q = 2\pi\int_{Rc}^{R} r\,u\,dr$$

$$= -\frac{\pi R_0^4\left\{(R/R_0)^2 - \varepsilon^2\right\}}{8\mu}\frac{dp}{dz}\left[(R/R_0)^2 + \varepsilon^2 - \frac{\left\{(R/R_0)^2 - \varepsilon^2\right\}}{\log\{(R/R_0)/\varepsilon\}}\right], \quad (8)$$

where $\varepsilon = R_c/R_0$.

From equation (8), one now obtains

$$-\frac{dp}{dz} = \frac{8\mu\,Q}{\pi R_0^4}\varphi(z), \quad (9)$$

with $\varphi(z) = 1/F(z)$, $F(z) = \left\{(R/R_0)^2 - \varepsilon^2\right\}\left[(R/R_0)^2 + \varepsilon^2 - \frac{\left\{(R/R_0)^2 - \varepsilon^2\right\}}{\log\{(R/R_0)/\varepsilon\}}\right]$.

The pressure drop, $\Delta p\,(= p \text{ at } z = 0, - p \text{ at } z = L)$ across the stenosis in the tube of length, L is obtained as

$$\Delta p = \int_0^L\left(-\frac{dp}{dz}\right)dz$$

$$= \frac{8\mu Q}{\pi R_0^4}\psi, \quad (10)$$

where

$$\psi = \int_0^d[\varphi(z)]_{R/R_0=1}dz + \int_d^{d+L_0/2}[\varphi(z)]_{R/R_0\ from\ (1)}dz + \int_{d+L_0/2}^{d+L_0}[\varphi(z)]_{R/R_0\ from\ (2)}dz + \int_{d+L_0}^{L}[\varphi(z)]_{R/R_0=1}dz$$

The first and the fourth integrals in the expression for ψ obtained above are straight forward whereas the analytical evaluation of second and third integrals are almost a formidable task and therefore shall be evaluated numerically. Following now the definitions given in Srivastava and Rastogi (2009, 2010), one derives the expressions for the impedance (flow resistance), λ, the wall shear stress distribution in the stenotic region, τ_w and shear stress at the stenosis throat, τ_s in their non-dimensional form as

$$\lambda = \frac{1 - L_0/L}{\eta} + \frac{1}{L}\int_d^{d+L_0/2}\frac{dz}{(a^2 - \varepsilon^2)\left[a^2 + \varepsilon^2 - (a^2 - \varepsilon^2)/\log(a/\varepsilon)\right]}$$

$$+\frac{L_0}{2\pi L}\int_0^{\pi}\frac{d\alpha}{(\theta^2 - \varepsilon^2)\left[\theta^2 + \varepsilon^2 - (\theta^2 - \varepsilon^2)/\log(\theta/\varepsilon)\right]}, \quad (11)$$

$$\tau_w = \frac{R/R_0}{\left\{(R/R_0)^2 - \varepsilon^2\right\}\left[(R/R_0)^2 + \varepsilon^2 - \left\{(R/R_0)^2 - \varepsilon^2\right\}/\log((R/R_0)/\varepsilon)\right]}, \quad (12)$$

$$\tau_s = \frac{b}{\{b^2-\varepsilon^2\}\left[b^2+\varepsilon^2-\{b^2-\varepsilon^2\}/\log(b/\varepsilon)\right]}, \tag{13}$$

where

$$a \approx a(z) = 1-2(\delta/R_0)(z-d)/L_0\ ,$$
$$b = 1-\delta/R_0, \qquad c = -\delta/2R_0\,,$$
$$\theta \approx \theta(\alpha) = b + c\cos\alpha, \qquad \alpha = \pi-(2\pi/L_0)(z-d-L_0/2),$$
$$\eta = (1-\varepsilon^2)\{1+\varepsilon^2+(1-\varepsilon^2)/\log\varepsilon\},$$
$$\lambda = \bar{\lambda}/\lambda_0,\ \ (\tau_w, \tau_s) = (\bar{\tau}_w, \bar{\tau}_s)/\tau_0\,,$$

$\lambda_0 = 8\mu L/\pi R_0^4$, $\tau_0 = 4\mu Q/\pi R_0^3$ are the flow resistance and shear stress, respectively for a Newtonian fluid in a normal artery (no stenosis), and $\bar{\lambda}$, $\bar{\tau}_w$ and $\bar{\tau}_s$ are the impedance, wall shear stress and shear stress at stenosis throat, respectively in their dimensional form obtained from the definitions: $\bar{\lambda} = \Delta p/Q$, $\bar{\tau}_w = (-R/2)dp/dz$, $\bar{\tau}_s = (\bar{\tau}_w)_{R/R_0=b}$.

NUMERICAL RESULTS AND DISCUSSION

To discuss the results of the study quantitatively, computer codes are now developed for the numerical evaluations of the analytical results obtained in equations (11)-(13). The various parameter values are selected from Young (1968), Srivastava and coworkers (2009, 2010) as: L_0 (cm) = 1; L (cm) = 1, 2, 5; ε (non-dimensional catheter radius) = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6; δ/R_0 (non-dimensional stenosis height) = 0, 0.05, 0.10, 0.15, 0.20. It is to note here that the present study corresponds to the flow in uncatheterized and normal (no stenosis) artery for parameter values ε=0 and δ/R_0= 0, respectively. The impedance, λ increases with the catheter size, ε for any given stenosis height, δ/R_0 and also increases with stenosis height, δ/R_0 for any given catheter size, ε (Fig.2).

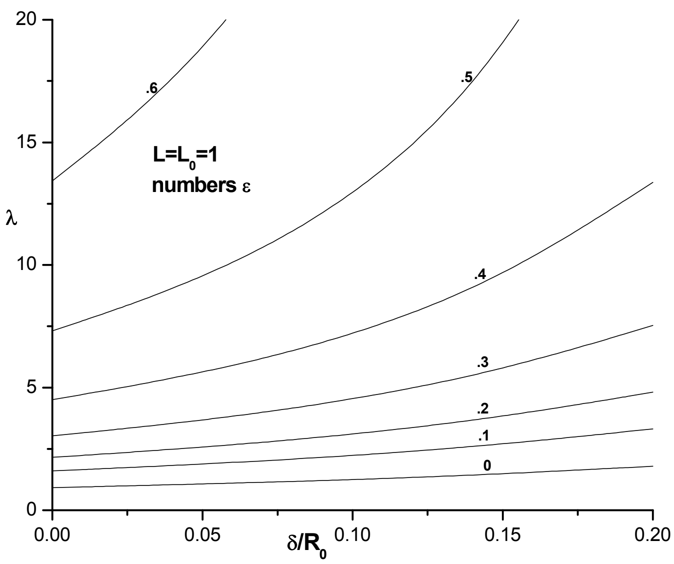

Fig.2 Variation of impedance,λ with $\delta/R_0$for different ε.

One notices that for any given stenosis height, a significant increase in the magnitude of the flow resistance, λ occurs for any small increase in the catheter size, ε (Fig.2). Numerical results reveal that for any given set of other parameters, the impedance, λ decreases with increasing the tube length, L which interns implies that the flow resistance, λ increases with the stenosis length, L_0. The flow resistance, λ steeply increases with the catheter size, ε (≤ 0.3) but increases rapidly with increasing the catheter size, ε and depending on the height of the stenosis, attains a very high asymptotic magnitude with increasing the catheter size, ε (Fig.3).

The high asymptotic magnitude of λ occurs for $\delta/R_0 = 0.1$(19% stenosis), 0.15 (28% stenosis) and 0.2 (38% stenosis) at catheter size, ε = 5.5, 4.5 and 4.0, respectively.

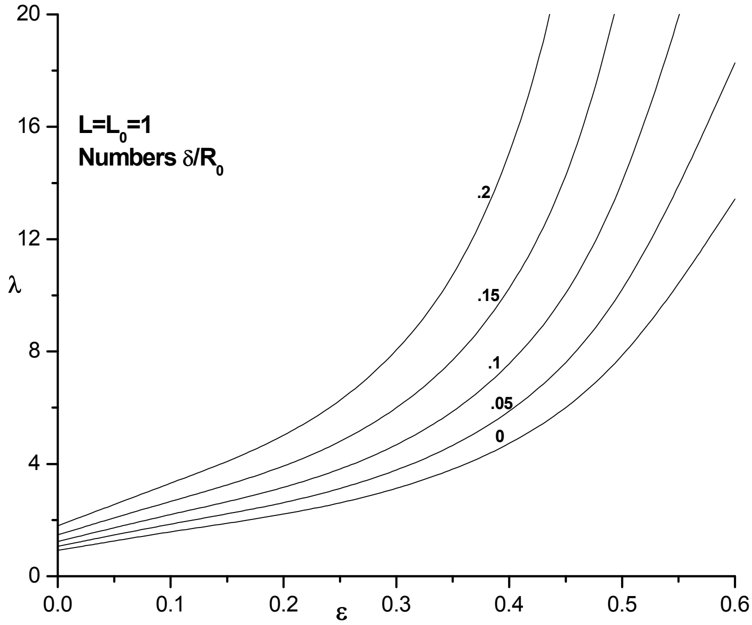

Fig.3 Variation of impedance, λ with ε for different δ/R_0.

The wall shear stress in the stenotic region, τ_w increases rapidly with the axial distance z/L_0 in the upstream of the stenosis throat and attains its peak value at the throat (i.e., at $z/L_0 = 1/2$). It then decreases in the down stream of the throat and achieves its approached value (i.e., at z = 0) at the critical axial distance, $z/L_0 = 7\,{}^3/_4$. The flow characteristic, τ_w further decreases in the range of the axial values: $7\,{}^3/_4 \leq z \leq 1$ and attains reasonably lower magnitude at the end point of the constriction profile (i.e., at $z/L_0 = 1$) than its approached value at z = 0 (Fig. 4). It is interesting to note that the blood flow characteristic, τ_w increases with stenosis height, δ/R_0 in the region $0 \leq z/L_0 \leq 7\,{}^3/_4$ but the property reverses in the region $7\,{}^3/_4 \leq z/L_0 \leq 1$ (Fig.4). The flow characteristic, τ_w increases with the catheter size, ε at any axial distance in the stenotic region. For small catheter size, ($\leq$ 0.3), the variations (increase or decrease) in the magnitude of τ_w seems to be steeply, however, for larger values of ε, the magnitude of the flow characteristic τ_w varies rapidly (Fig.5). The shear stress at the stenosis throat, τ_s (at $z/L_0 = ½$) possesses the characteristics similar to that of the flow resistance, λ with respect to any parameter.

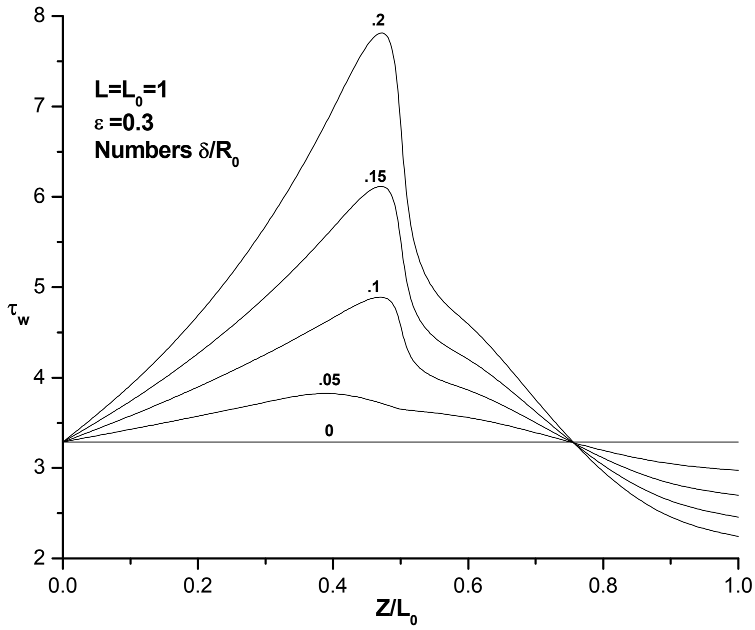

Fig.4 Wall shear stress distribution in the stenotic region for different δ/R_0.

However, the magnitude of the shear stress, τ_s is noted to be reasonably higher than the corresponding magnitude of the impedance, λ for any given set of parameters (Figs. 2, 3 and 6, 7).

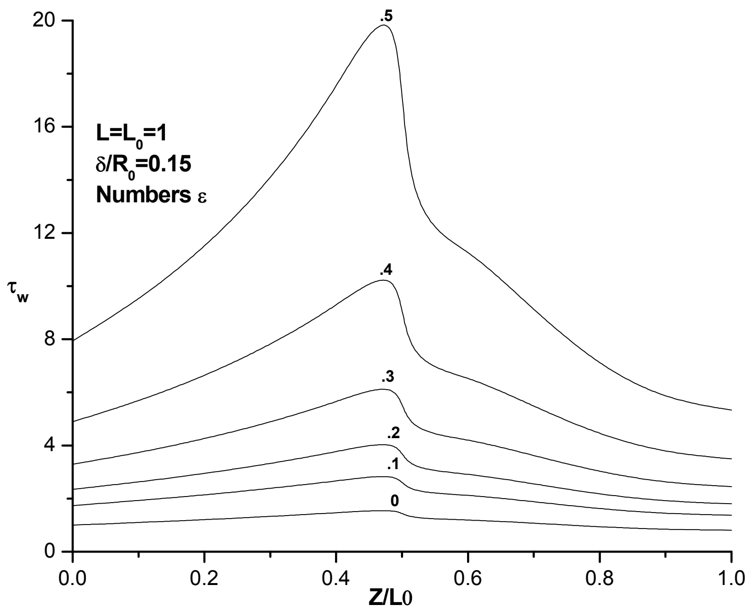

Fig.5 Wall shear stress distribution in the stenotic region for different ε.

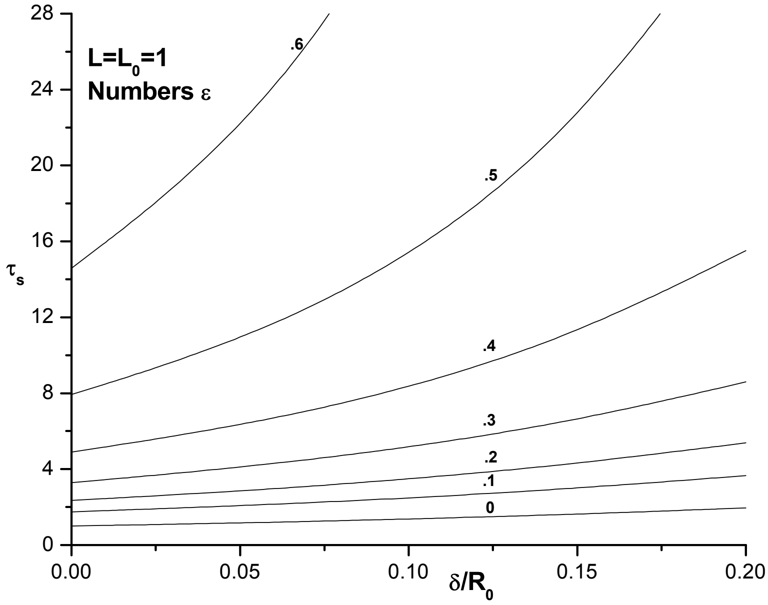

Fig.6 Variation of shear stress at stenosis throat, τ_s with δ/R_0 for different ε.

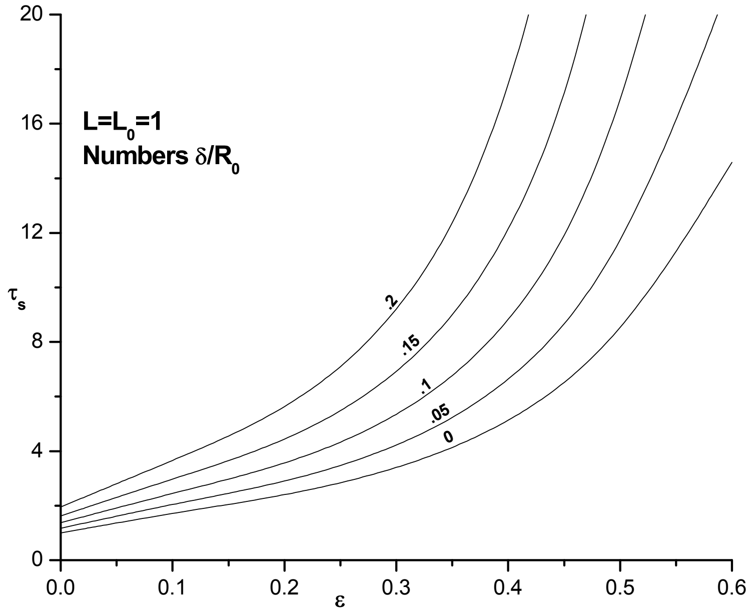

Fig.7 Variation of shear stress at stenosis throat, τ_s with ε for different δ/R_0.

CONCLUSIONS

To estimate for the increased impedance and shear stress during artery catheterization, flow through a composite stenosis has been analyzed assuming that the flowing blood is represented by a Newtonian fluid. The impedance increases with increasing catheter size and strongly depends on the stenosis height is an important information. Thus the size of the catheter must be chosen keeping in view of stenosis height during the medical treatment. At the critical axial distance in the stenotic region shear stress achieves the approached value which may be viewed as a special feature of the composite stenosis. In addition, shear stress decreases with increasing stenosis height in the region between the critical point and the end point of the constriction profile and its magnitude at the end point of the constriction profile assumes much lower value than its approached one may be noted as points of particular interest of the present work.

The significance of the present analysis is clearly understood from the above discussion and the conclusion. The condition: $\delta / R_0 << 1$, limits the usefulness of the study to very early stages of the vessel constriction, which enables one the use of fully developed flow equations and leads to the locally Poiseuille like flow and closed form solutions. Parameter δ / R_0 is restricted up to 0.15(i.e., 28% stenosis by area reduction) as beyond this value a separation in the flow may occur even at a relatively small Reynolds number (Young, 1968; Srivastava, 1995). The consideration of a pulsatile flow and the cases of a severe stenosis, however, are the future scope of the study. Further careful investigations are thus suggested to address the problem more realistically and to overcome the restrictions imposed on the present work.

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