Study of Heat Transfer during Peristaltic Transport of Fractional Second Grade Fluid through Non -uniform tube with permeable wall in different wave forms

Siddharth Shankar Bhatt[a], Amit Medhavi[b], R. S. Gupta[c], U.P. Singh[d]

a. Department of Applied Sciences and Humanities, Kamla Nehru Institute of Technology, Sultanpur, India
b. Department of Mechanical Engineering, Kamla Nehru Institute of Technology, Sultanpur, India
c. Department of Applied Sciences and Humanities, Institute of Engineering and Technology, Lucknow, India
d. Department of Mathematics, GLA University, Mathura, India

Telephone no-8005336482 Email Id- shankarbhatt56@gmail.com

ABSTRACT

The present study is concerned with heat transfer during peristaltic transport of fractional second grade fluid in a non-uniform cylindrical tube with permeable walls having different wave forms. The analysis is done under long wavelength and low Reynolds number approximation. The analytical solution for pressure gradient, frictional force and temperature are obtained for different wave forms. The effects of various parameters such as, non-uniformity of tube, permeability of walls, heat source/sink parameter and amplitude ratio on pressure rise, friction force and temperature are discussed.

Keywords: Peristalsis, Permeable wall, Non-Uniform Tube, Heat Transfer, Fractional second grade model.

Introduction

Peristalsis is a mechanism for pumping of physiological and industrial fluids. According to biological point of view, it is a neuromuscular property. Peristaltic pumping occurs when progressive waves of area contraction propagate along the length of distensible duct. In physiology, peristalsis occurs during urine transport from kidney to bladder through ureter, in movement of ovum in the fallopian tubes, in movement of chyme in gastrointestinal tract, in vasomotion of small blood vessels and in transport of spermatozoa in ducts efferentes of male reproductive tracts. Finger and roller pump based on the mechanism of peristaltic transport are used to pump corrosive or pure material.

First theoretical investigation of peristalsis using fluid mechanics principle is done by Latham [1]. Shapiro et.al [2] analyzed the significance of different parameters involved in peristaltic motion in. Victor & Shah [3] analyzed Heat transfer to blood flowing in a tube in. Recently Srinivas and Kothandapani [4] in investigated peristaltic transport in an asymmetric channel with heat transfer. Most of the physiological vessels are of nonuniform geometry therefore some researchers have taken non-uniform geometry for analysis such as Radhakrishnamacharya and Radhakrishna [5] discussed Heat transfer to peristaltic transport in a non-uniform channel. Prasanna Hariharan et.al [6] discuss peristaltic transport of non-newtonian fluid in diverging tube with different wave form. Nadeem and Akbar [7] studied influence of heat transfer on a peristaltic transport of

Herschel-Bulkley fluid in a non- uniform tube. Ellahi et. al [8] discussed effect of heat and mass transfer on peristaltic flow in non-uniform rectangular duct.
Most of the physiological fluid shows viscous and elastic property. Now a days, fractional calculus is widely used to discuss viscoelastic nature of the fluid. In this continuation, fractional second grade model is applied to study of movement of chyme through small intestine and esophagus. Some viscoelastic model have been studied by different researchers such as periodic unidirectional flow of unidirectional flows of viscoelastic fluid with fractional Maxwell model have been studied by Hayat et. al [9], peristaltic flow of viscoelastic fluid with fractional Maxwell model through a channel have been investigated by Tripathi et.al [10], peristaltic transport of a generalized Burgers fluid: application to movement of chime in small intestine is discussed by Tripathi. et.al [11]. Tripathi. [12] has analyzed peristaltic transport of viscoelastic fluid through cylindrical tube. Narala et.al [13] have studied peristaltic motion of viscoelastic fluid with fractional second grade model in curved channel. Rathod. et al., [14] have studied interaction of heat transfer and peristaltic pumping of a fractional second grade fluid through a vertical cylindrical tube. Hameed et. al [15] investigated magnetic and heat transfer on peristaltic transport of a fractional second grade fluid in a vertical tube.
The objective of the present paper is to compare the effect of various parameters in different form of wave propagation on heat transfer during peristaltic flow through non-uniform tube with permeable walls. This problem is analytically solved by use of fractional calculus. Under long wavelength and low Reynolds number approximation are taken.

Some necessary definition

Definition1. The Riemann-Liouville fractional integral operator of order $\alpha > 0$ of a function $f(x): (0, \infty) \rightarrow R$ is given by

$$J^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-\xi)^{\alpha-1} f(\xi) d\xi, \qquad \alpha > 0, x > 0 \tag{1}$$

Definition 2. The fractional derivative of order $\alpha > 0$ of a continuous function $f(x): (0, \infty) \rightarrow R$ is given by

$$D^{\alpha} f(x) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{d}{dx}\right)^m \int_0^x (x-\xi)^{m-\alpha-1} f(\xi) d\xi, \tag{2}$$

for $\quad m-1 < \alpha \leq m, m \in N, x > 0, f \in C_{-1}^m,$

where $m = [\alpha] + 1$, provided that right- hand side is pointwise defined on $(0, \infty)$.
Remark 3. For example, $f(x) = x^{\beta}$, we quote for $\beta > -1$, in (2) one can get

$$D^{\alpha} x^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} x^{\beta-\alpha}, \tag{3}$$

giving in particular $D^{\alpha} x^{\beta-n} = 0, n = 1,2,3, \ldots . N,$ Where N is the smallest integer greater than or equal to α.

Mathematical Model

Consider the flow of an incompressible fractional second grade fluid due to peristaltic transport induced by sinusoidal wave trains propagating with constant speed c. The temperatures of walls of tube are T_o. The constitutive equation for viscoelastic fluid with fractional second grade model is given by:

$$\tilde{s} = \mu\left(1 + \tilde{\lambda}_1{}^{\alpha}\frac{\partial^{\alpha}}{\partial \tilde{t}^{\alpha}}\right)\dot{\gamma} \tag{4}$$

where $\tilde{t}, \tilde{S}, \dot{\gamma}$ and $\tilde{\lambda}_1$ is time, shear stress, rate of shear strain and material constant, respectively, $\mu -$ is viscosity, and $a -$ is fractional time derivative parameters such that $0 < a \leq 1$. This model reduces to second grade models when $a = 1$ and classical Navier-Stokes model is obtained by substituting $\tilde{\lambda}_1 = 0$.

The governing equations of the motion for viscoelastic fluid with fractional second grade model through tube for axisymmetric flow are given by:

$$\left.\begin{aligned}
&\frac{1}{R}\frac{\partial(RV)}{\partial R} + \frac{\partial U}{\partial X} = 0\\
&\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial R}\right) = -\frac{\partial p}{\partial X} + \mu\left(1 + \tilde{\lambda}_1{}^{\alpha}\frac{\partial^{\alpha}}{\partial \tilde{t}^{\alpha}}\right)\left[\frac{1}{R}\frac{\partial\left(R\frac{\partial U}{\partial R}\right)}{\partial R} + \frac{\partial^2 U}{\partial X^2}\right] + \rho g \alpha_1 (T - T_O)\\
&\rho\left(\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial R}\right) = -\frac{\partial p}{\partial R} + \mu\left(1 + \tilde{\lambda}_1{}^{\alpha}\frac{\partial^{\alpha}}{\partial \tilde{t}^{\alpha}}\right)\left[\frac{1}{R}\frac{\partial(RV)}{\partial R} + \frac{\partial^2 V}{\partial X^2}\right]\\
&\rho c_p\left(\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial R}\right) = K\left(\frac{\partial^2 T}{\partial R^2} + \frac{1}{R}\frac{\partial T}{\partial R} + \frac{\partial^2 T}{\partial X^2}\right) + Q_O
\end{aligned}\right\} \tag{5}$$

Introducing non-dimensional parameters

$$V' = \frac{V}{c\delta}, U' = \frac{U}{c}, X' = \frac{X}{\eta}, R' = \frac{R}{a}, \lambda' = \frac{\widetilde{\lambda_1} c}{\eta}, \delta = \frac{a}{\eta}, p' = \frac{pa^2}{\mu c \eta}, Re = \frac{\rho a c \delta}{\mu}$$

$$\theta = \frac{T - T_o}{T_O}, \quad Gr = \frac{\rho g \alpha_1 a^2 T_o}{\mu c}, \quad t' = \frac{ct}{\eta},$$

$$\beta = \frac{a^2 Q_o}{K T_o} \tag{6}$$

Where ρ is density of fluid, Q_o is the constant heat, $p, U, V, R, \mu, K, \alpha_1$ stands for pressure, axial velocity, radial velocity, radial coordinate, coefficient of viscosity, thermal conductivity, coefficient of expansion.

Using equation (6) in (5) and considering long wavelength and low Reynolds number approximation, after dropping primes, we have

$$\left.\begin{aligned}&\frac{\partial p}{\partial X}=\left(1+\lambda^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\left[\frac{1}{R}\frac{\partial\left(R\frac{\partial U}{\partial R}\right)}{\partial R}\right]+\theta Gr\\&\frac{\partial p}{\partial R}=0\\&\frac{\partial^{2}\theta}{\partial R^{2}}+\frac{1}{R}\frac{\partial\theta}{\partial R}+\beta=0\end{aligned}\right\}\tag{7}$$

With boundary conditions

$$\left.\begin{aligned}&\frac{\partial\theta}{\partial R}=0\quad at\ R=0,\quad \theta=0\quad at\quad R=h\\&\frac{\partial U}{\partial R}=0\quad at\ R=0,\quad U=-k\frac{\partial U}{\partial R}\quad at\quad R=h\end{aligned}\right\}\tag{8}$$

On solving equation (7) with boundary condition (8), we get

$$\left(1+\lambda^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)U=\left(\frac{R^{2}}{4}-\frac{h^{2}}{4}-\frac{kh}{2}\right)\frac{\partial p}{\partial X}-\frac{Gr\beta}{64}\left(4R^{2}h^{2}-R^{4}-3h^{4}-4kh^{3}\right)\tag{9}$$

$$\theta=\frac{\beta}{4}\left(h^{2}-R^{2}\right)\tag{10}$$

The volumetric flow rate is thus calculated as

$$\bar{Q}=\int_{0}^{h}2\pi RUdR\tag{11}$$

Using equation (9), we get

$$\left(1+\lambda^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\bar{Q}}{\pi}=\left(-\frac{h^{4}}{8}-\frac{kh^{3}}{2}\right)\frac{\partial p}{\partial X}+\frac{Gr\beta kh^{5}}{16}+\frac{2Gr\beta h^{6}}{96}$$

or

$$\frac{\partial p}{\partial X}=\frac{\left(1+\lambda^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\bar{Q}}{\pi}-\frac{2Gr\beta h^{6}}{96}-\frac{Gr\beta kh^{5}}{16}}{\left(-\frac{h^{4}}{8}-\frac{kh^{3}}{2}\right)}\tag{12}$$

Pressure rise and friction force at wall are given by

$$\Delta p\int_{0}^{1}\frac{dp}{dX}dX\tag{13}$$

$$F\int_{0}^{1}h^{2}\left(-\frac{dp}{dX}\right)dX\tag{14}$$

Expressions for wall shape, h(X,t), for different wave forms are tabulated as [6]

Wave form	Expression for h(X,t)
Sinusoidal	$1 + BX + \emptyset \sin 2\pi(X - t)$
Triangular	$1 + BX + \emptyset\left[\frac{8}{\pi^3}\sum_1^\infty \frac{(-1)^{n+1}}{(2n-1)^2}\sin\{(2n-1)2\pi(X-t)\}\right]$
Trapezoidal	$1 + BX + \emptyset\left[\frac{32}{\pi^2}\sum_1^\infty \frac{\sin\frac{\pi}{8}(2n-1)}{(2n-1)^2}\sin\{(2n-1)2\pi(X-t)\}\right]$
Square	$1 + BX + \emptyset\left[\frac{4}{\pi}\sum_1^\infty \frac{(-1)^{n+1}}{(2n-1)}\cos\{(2n-1)2\pi(X-t)\}\right]$

Expressions for instantaneous flow rate, Q(X,t), for different wave forms [6]

Wave form	Expression for Q(X,t)
Sinusoidal	$\frac{\bar{Q}(X,t)}{\pi} = \frac{Q}{\pi} - \frac{\emptyset^2}{2} + (1 + BX)2\emptyset \sin 2\pi(X-t) + \emptyset^2 sin^2\{2\pi(X-t)\}$
Triangular	$\frac{\bar{Q}(X,t)}{\pi} = \left[\frac{Q}{\pi} - \left(\frac{32\emptyset^2}{\pi^4}\right)\sum_1^\infty\left\{\frac{1}{(2n-1)^4}\right\}\right] + \left[\frac{16\emptyset}{\pi^2}(1 + BX)\sum_1^\infty\left\{\frac{(-1)^{n+1}}{(2n-1)^2}\right\}\sin\{(2n-1)2\pi(X-t)\}\right] + \left[\frac{64\emptyset^2}{\pi^2}\sum_1^\infty\left\{\frac{1}{(2n-1)^4}\right\} sin^2\{(2n-1)2\pi(x-t)\}\right]$

Trapezoidal	$\frac{\bar{Q}(X,t)}{\pi} = \left[\frac{Q}{\pi} - \left(\frac{512\emptyset^2}{\pi^4}\right)\sum_1^\infty \frac{\left(\sin\frac{\pi}{8}(2n-1)\right)^2}{(2n-1)^4}\right] + \left[\frac{64\emptyset}{\pi^2}(1 + BX)\sum_1^\infty \left\{\frac{\sin\frac{\pi}{8}(2n-1)}{(2n-1)^2}\right\}\sin\{(2n-1)2\pi(X - t)\}\right] + \left[\left(\frac{1024}{\pi^4}\right)\emptyset^2\sum_1^\infty\left\{\frac{\left(\sin\frac{\pi}{8}(2n-1)\right)^2}{(2n-1)^4}\right\} sin^2\{(2n - 1)2\pi(X - t)\}\right]$
Square	$\frac{\bar{Q}(X,t)}{\pi} = \left[\frac{Q}{\pi} + \left(\frac{8\emptyset^2}{\pi^2}\right)\sum_1^\infty\left\{\frac{1}{(2n-1)^2}\right\}\right] + \left[\frac{8\emptyset}{\pi}(1 + BX)\sum_1^\infty \frac{(-1)^{n+1}}{(2n-1)}\cos\{(2n-1)2\pi(X-t)\}\right] + \left[\left(\frac{16}{\pi^2}\right)\emptyset^2\sum_1^\infty\left\{\frac{1}{(2n-1)^2}\right\}cos^2\{(2n-1)2\pi(X - t)\}\right]$

Results and Discussion

MATHEMATICA package is used to estimate the effects of various parameter involved in the result of the present study. Lots of computation has been performed to reveal the influence of geometry, hydrodynamical parameters on pressure, frictional force and heat transfer. Parameters analyzed are non-uniformity of the geometry B, amplitude ratio $\emptyset$, heat source/sink parameter β, permeability k in case of different wave forms. The numerical evaluations of the analytical results are obtained for $\Delta P, f\ and\ \theta$. Figures 1-4 are plotted to see the variation of pressure rise for different physical parameters in different wave forms. In figures 1- 4 all graphs corresponding to α = 1 represent second grade fluid. It is examined from figures 1-4 that there exists direct proportionality between pressure rise Δp and flow rate Q irrespective of the parameter varied. Figure 1 shows the fact that pressure increases with increase in source/sink parameter β for fixed flow rate in different wave forms. It is also observed that pressure rise for second grade fluid is less than that of fractional second grade fluid in different wave forms. Maximum pressure rise is obtained in case of trapezoidal wave for a particular value of source/sink parameter β at zero flow rate for fractional second grade fluid. Figure 2 shows the effect of change in non-uniformity parameter B on pressure in different wave form. It is observed that pressure rise decreases with increases in non-uniformity parameter B at zero flow rate for second grade as well as fractional second grade fluid in different wave forms except triangular wave. For triangular wave form, pressure rise decreases for fractional second grade fluid but increases for second grade fluid at zero flow rate with increase in non-uniformity parameter B. Maximum pressure rise is obtained in case of trapezoidal wave for a particular value of non-uniformity parameter B at zero flow rate for fractional second grade fluid. The variation of pressure against flow rate for various values of amplitude ratio ∅ is portray in figure 3 for different wave forms. From figure 3, it is revealed that on increasing the value of ∅ pressure rise increases for second grade fluid as well as fractional second grade fluid in different wave forms. Maximum pressure rise is obtained in case of trapezoidal wave for a particular value of ∅ at zero flow rate for fractional second grade fluid. Figure 4 shows variation in pressure with change in permeability k. It is evident from the figure 4 that pressure decrease with increase in k for given flow rate in different wave forms for second grade as well as fractional second grade fluid. Maximum pressure rise is obtained in case of trapezoidal wave at zero flow rate for fractional second grade fluid.

Figures 5-8 are plotted to see the variation of friction force against flow rate for different physical parameters in different wave forms. It is evident from figures 5-8 that there exists direct proportionality between frictional force F and flow rate Q irrespective of the parameter varied. Figure 5 depicts the variation of frictional force against flow rate for different values of source/sink parameter β. It can examine from figure 5 that frictional force decreases with increase in β at zero flow rate in different wave forms for second grade as well as fractional second grade fluid. Magnitude of frictional force for fractional second grade fluid is greater than that of second grade fluid at zero flow rate for different wave forms. Figure 6 shows the variation of frictional force against flow rate with change in non-uniformity parameter B. It is clear from the figure 6 that frictional force increases with increase in B for second grade as well as fraction second grade fluid. Magnitude of frictional force for fractional second grade fluid is greater than that of second grade fluid at zero flow rates in different wave forms. The effect of amplitude ratio ∅ is shown in figure 7. It is noted that frictional force decreases with increase in ∅ for a given flow rates. Magnitude of frictional force for fractional second grade fluid is greater than that of second grade fluid at zero flow rates for different wave forms for variation in ∅ respectively. Figure 8 disclosed the variation of frictional force against flow rate with change in permeability k. It can be seen that frictional force increases with increase in k for given flow rate in different wave forms. Magnitude of friction force for fractional second grade fluid is greater than that of second grade fluid at zero flow rates in different wave forms.

Figure 9 shows the effects of source/sink parameter β on temperature. Temperature increases with increase in source/sink parameter β at inlet as well as downstream in different wave forms. Maximum temperature at inlet and downstream is obtained for square and trapezoidal wave respectively. Figure 10 shows the effects of non-uniformity parameter B on temperature. Temperature increases with increase in non-uniformity parameter B downstream but remains unaffected at inlet in different wave forms. Maximum temperature at inlet and downstream is obtained for square and trapezoidal waves respectively. Fig 11 shows the effects of source/sink parameter ∅ on temperature. Temperature decreases with increase in amplitude ratio ∅ at inlet for different wave forms except for square wave and temperature increases with increase in amplitude ratio ∅ downstream in different wave forms except for square wave. Maximum temperature at inlet and downstream is obtained for square and trapezoidal waves respectively.

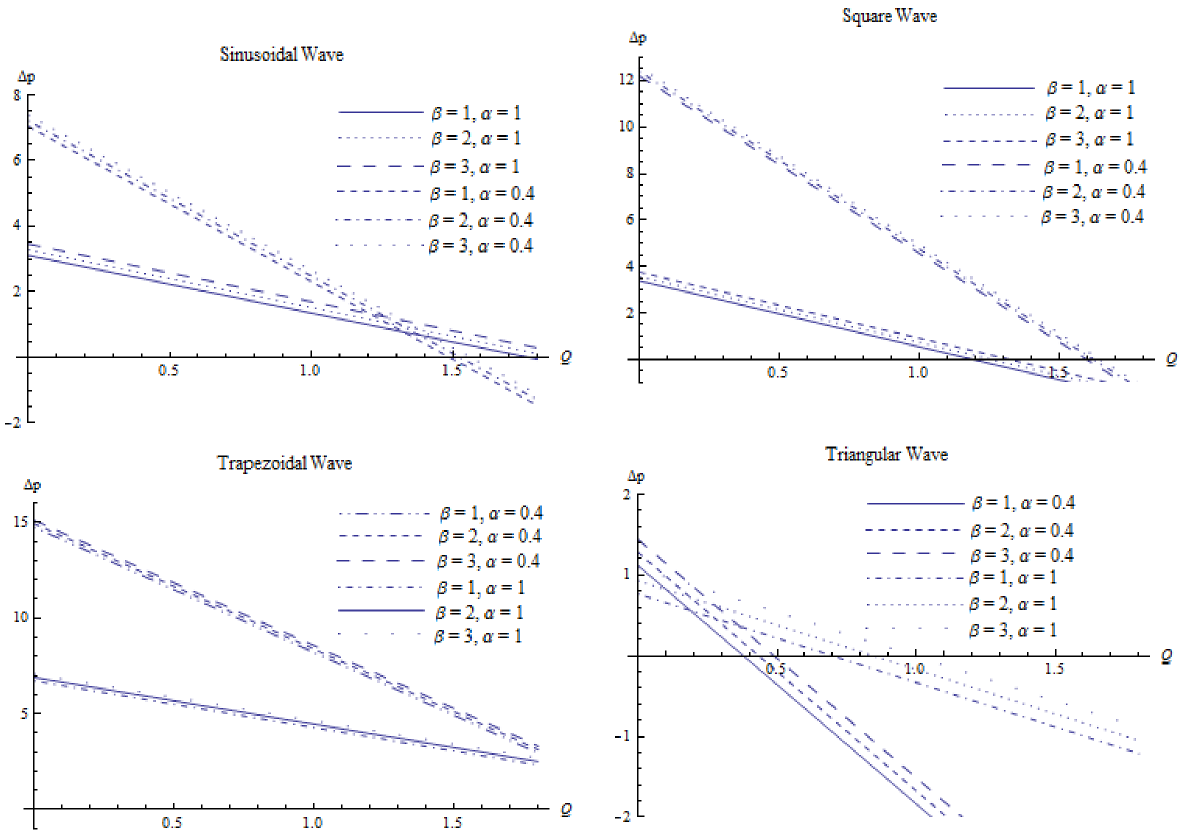
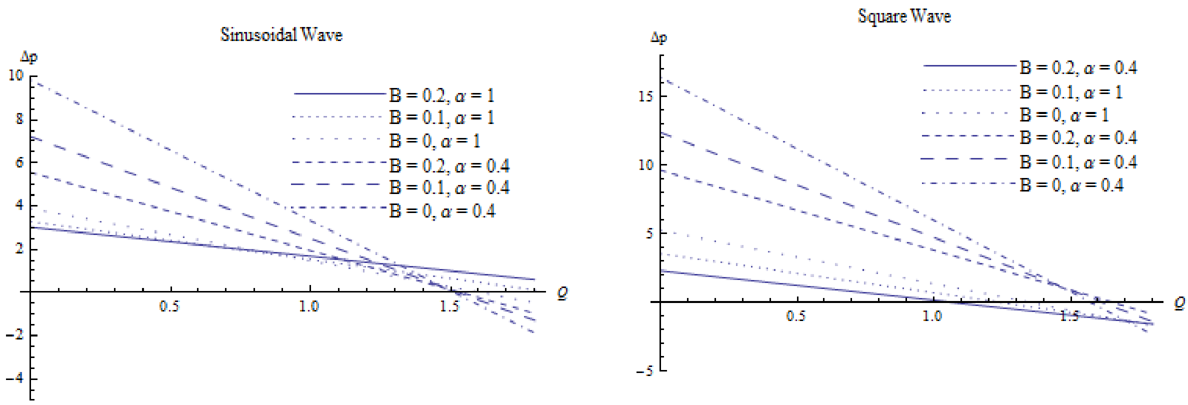


Figure 1: Pressure vs. averaged flow rate for various values of β at λ=1,B=0.1,t=0.1,φ=0.4,k=0.25,Gr=1,α=0.4 in different wave forms.



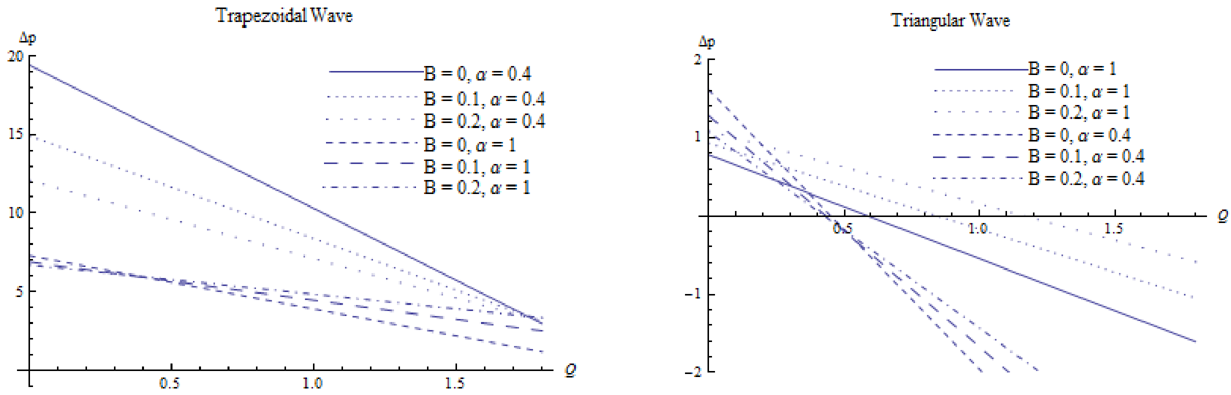


Figure 2: Pressure vs. averaged flow rate for various values of B at λ=1,α=0.4,t=0.1,φ=0.4,k=0.25,Gr=1,β=2 in different wave forms

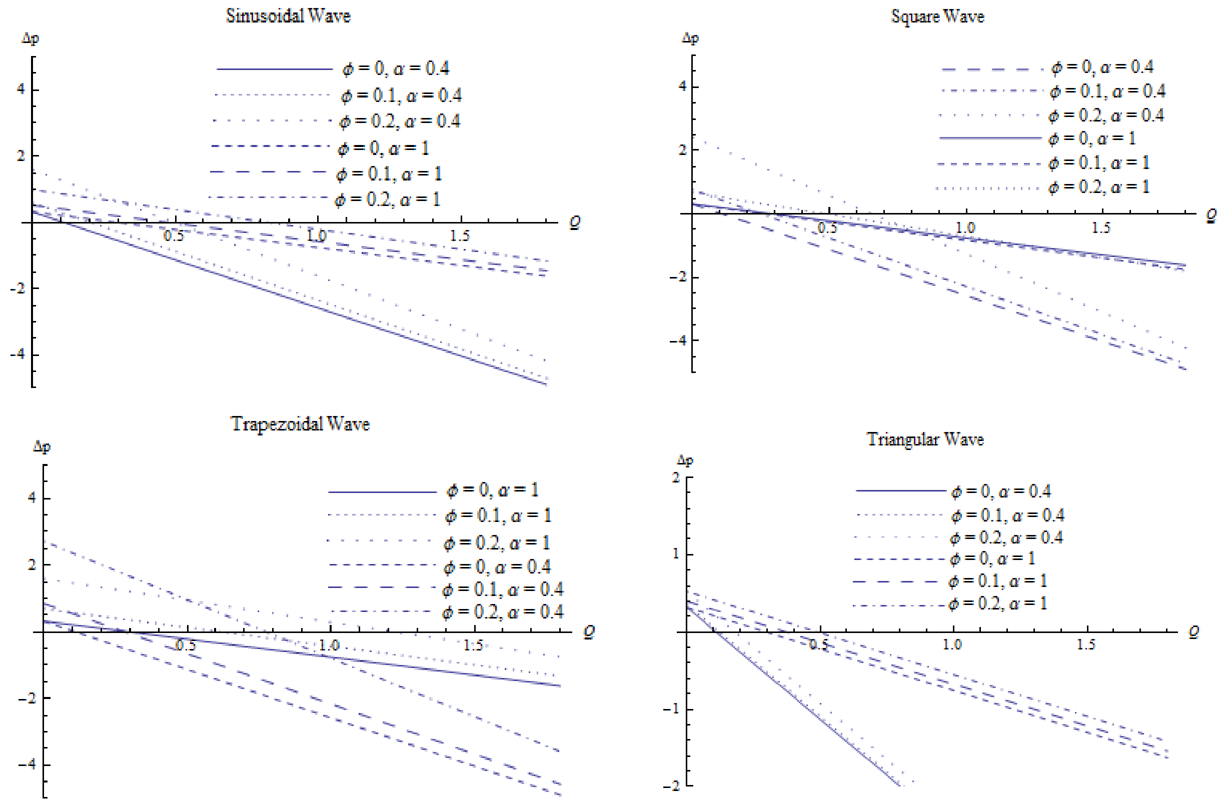


Figure 3: Pressure vs. averaged flow rate for various values of φ at λ=1,α=0.4,t=0.1,B=0.1,k=0.25,Gr=1,β=2 in different wave forms.

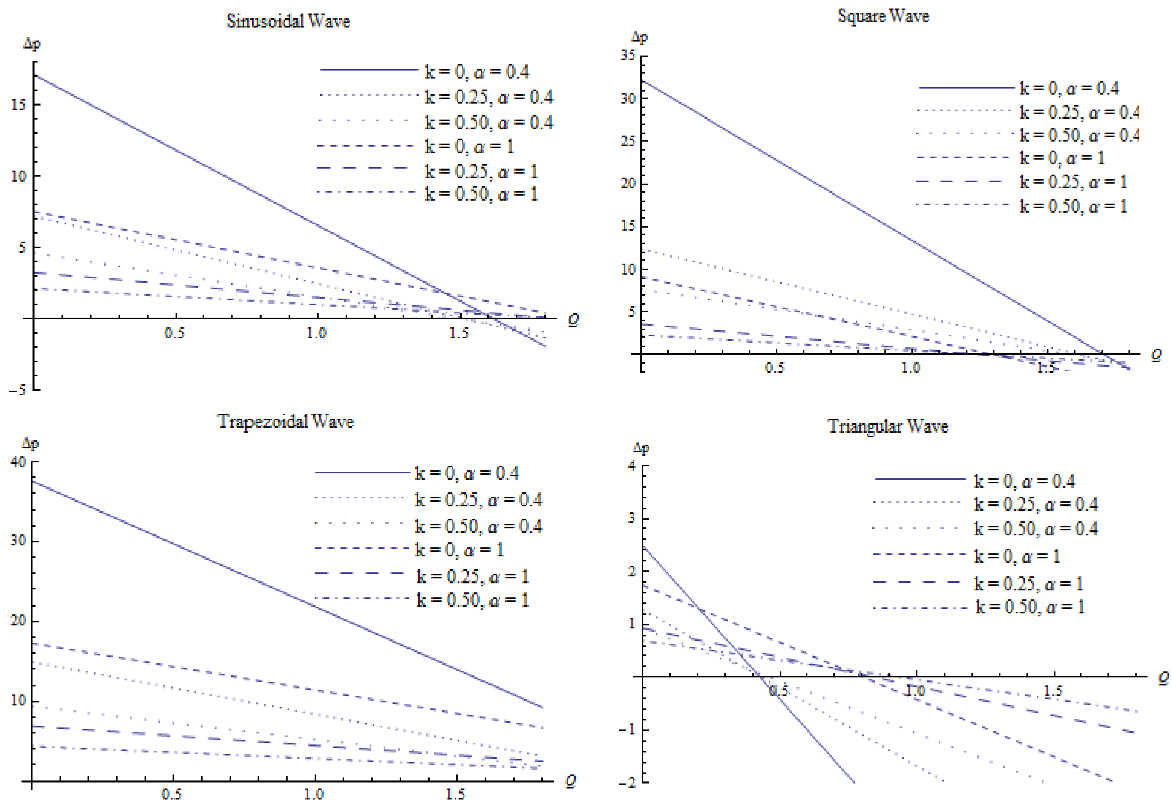


Figure 4: Pressure vs. averaged flow rate for various values of k at λ=1,α=0.4,t=0.1,φ=0.4,B=0.1,Gr=1,β=2 in different wave forms

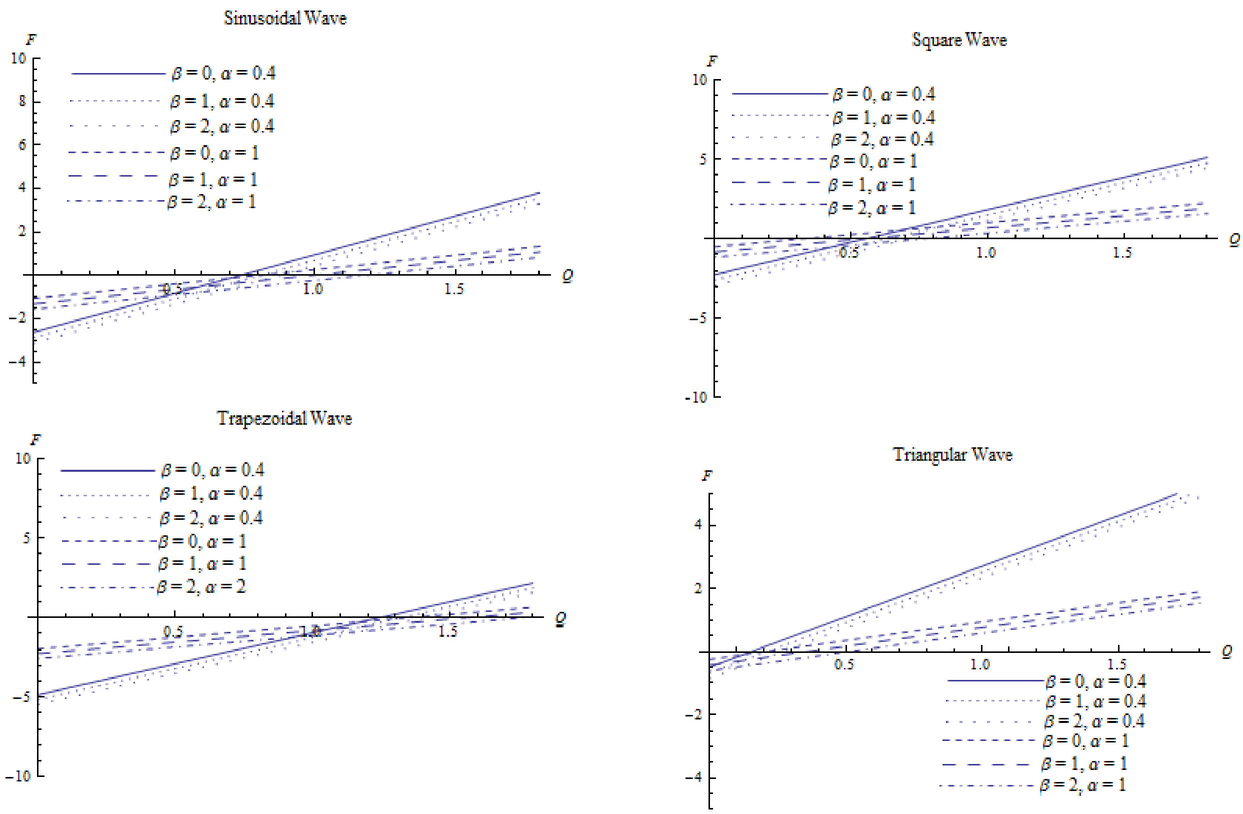


Figure 5: Frictional force vs. averaged flow rate for various values of β at λ=1,t=0.1,k=0.25,φ=0.4,B=0.1,Gr=1, α=0.4 in different wave forms

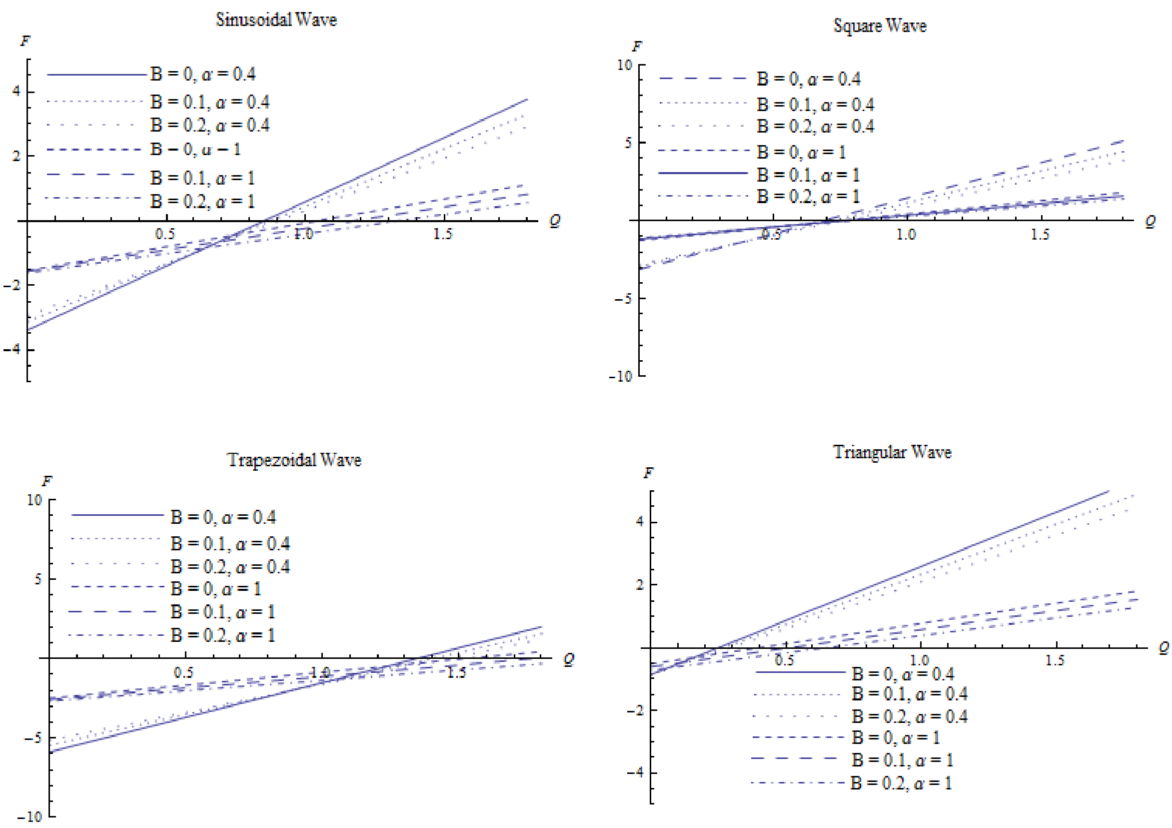


Figure 6: Frictional force vs. averaged flow rate for various values of B at λ=1,t=0.1,k=0.25,φ=0.4,Gr=1,α=0.4 β=2 in

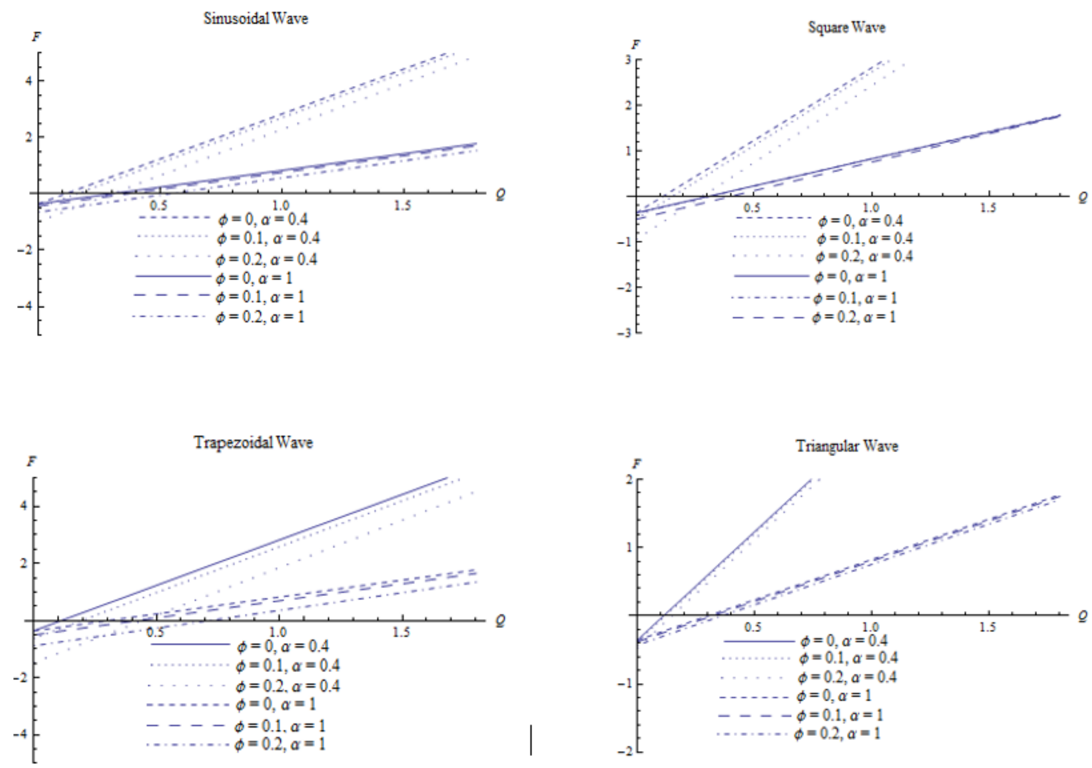


Figure 7: Frictional force vs. averaged flow rate for various values of φ at λ=1,t=0.1,k=0.25,Gr=1,B=0.1,α=0.4 β=2 in

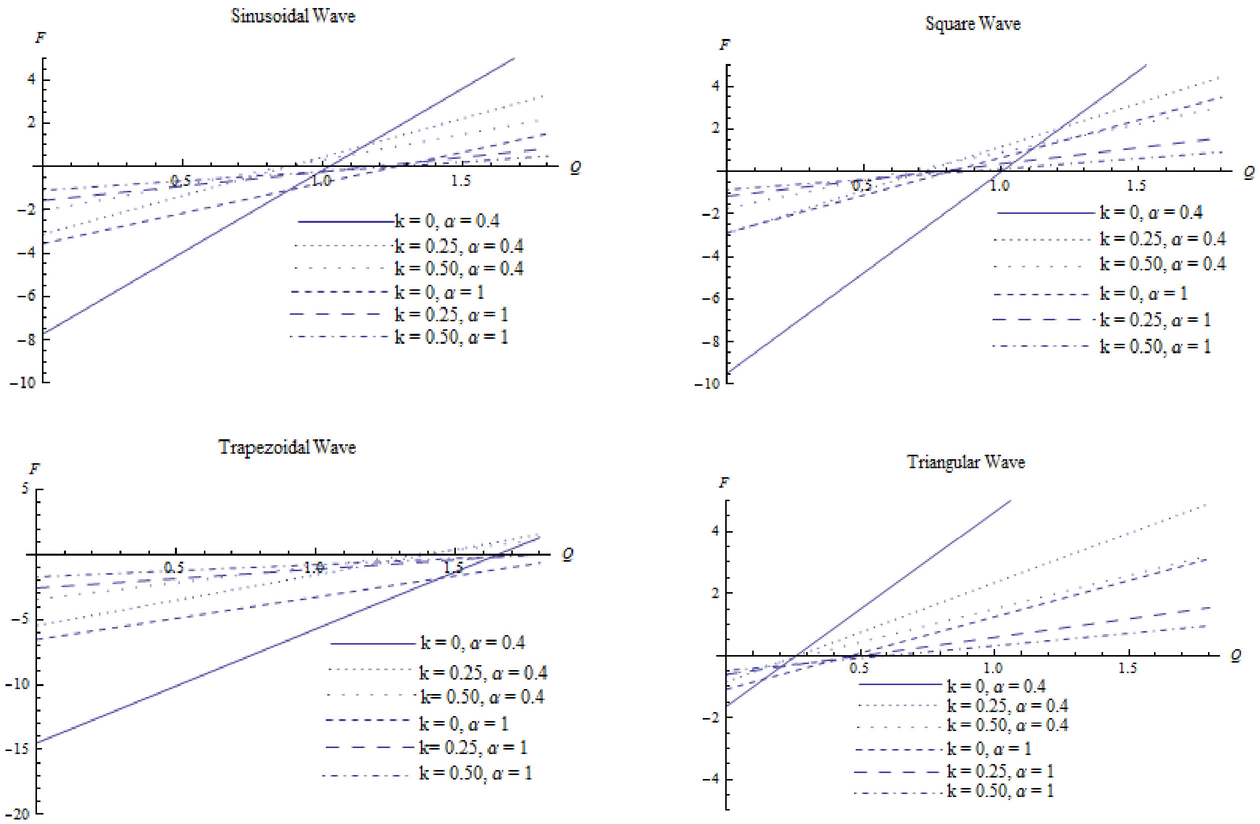


Figure 8: Frictional force vs. averaged flow rate for various values of k at λ=1,t=0.1,φ=0.4,Gr=1,B=0.1,α=0.4,β=2 in different wave forms.

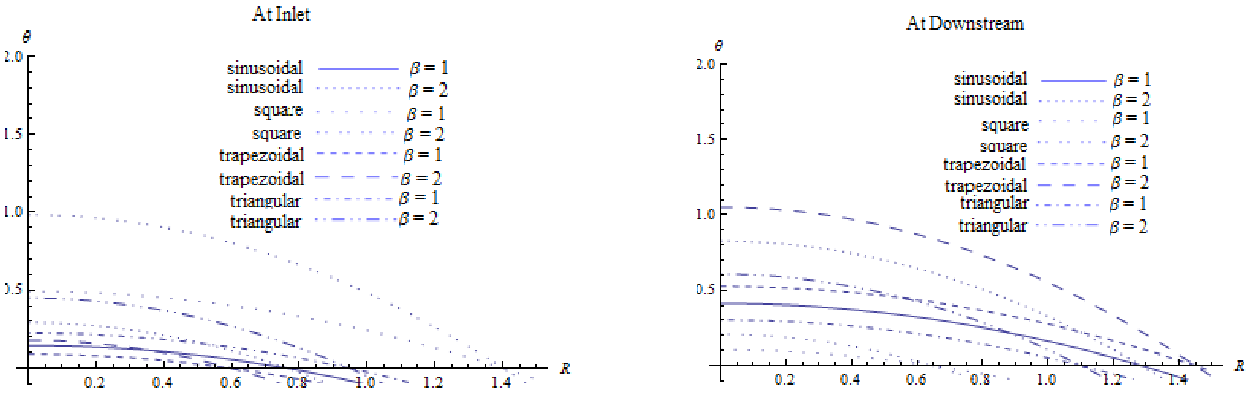


Figure 9 : Effect of β on temperature at φ=0.4, B=0.1,t=0.1 at inlet as well as downstream in different wave forms.

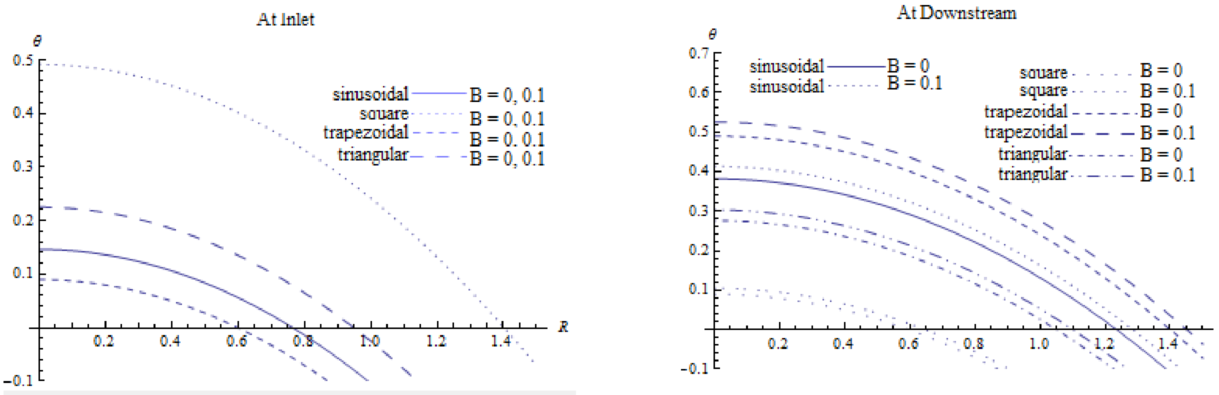


Figure 10 : Effect of B on temperature at β=1,φ=0.4,t=0.1 at inlet as well as downstream in different wave forms.

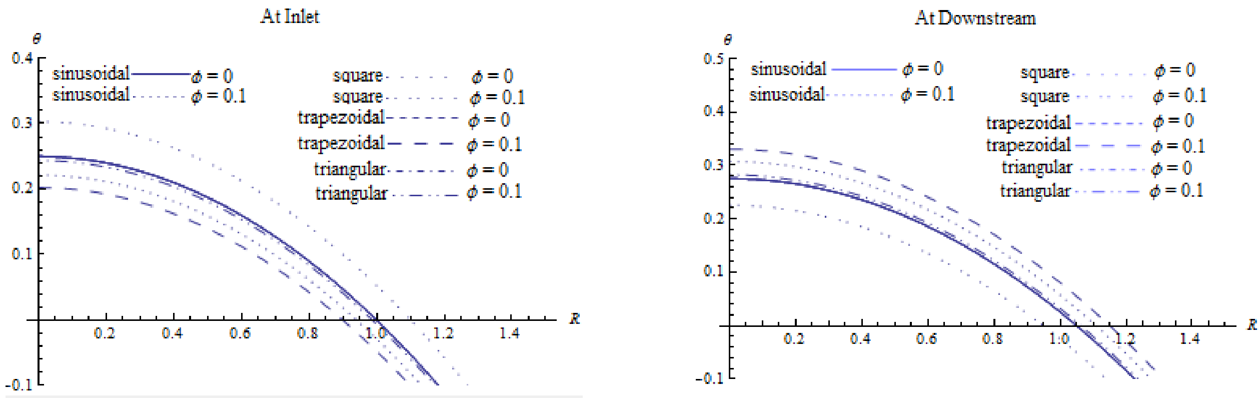


Figure 11: Effect of φ on temperature at β=1,B=0.1,t=0.1 at inlet as well as downstream in different wave forms.

Conclusion

In this paper, the problem of heat effect on viscoelastic fluid flow with fractional second grade model with peristaltic transport through non-uniform tube having permeable wall in different wave forms has been addressed. Expressions for pressure rise, frictional force and temperature are obtained under long wavelength and low Reynolds number approximations. Effect of non-uniformity of tube, permeability of wall, source/sink parameter, and amplitude ratio are studied. The following conclusion can be summarized.

(1) There exists linear relation between pressure rise and flow rate in different wave forms.
(2) There exists linear relation between frictional force and flow rate.
(3) The pressure rise increases with increasing value of source/sink parameter, amplitude ratio in different wave forms.
(4) At zero flow rate, pressure rise for fractional second grade fluid is greater than that of second grade fluid in different wave forms and maximum pressure rise is found in the case of trapezoidal wave.

(5) The variation of frictional force with flow rate shows opposite behavior to that of pressure rise.
(6) Temperature increases with increasing value of source/sink parameter at inlet as well as downstream of the tube in different wave forms. Maximum temperature at inlet of the tube is obtained for square wave whereas maximum temperature at downstream of the tube is obtained for trapezoidal wave.
(7) Temperature remains unchanged with increasing value of non-uniformity parameter at inlet in different wave form but increase as we move downstream. Maximum temperature at downstream of the tube is obtained for trapezoidal wave.
(8) Temperature decreases with increasing value of amplitude ratio at inlet in different wave forms except square wave and temperature increases with increasing value of amplitude ratio as we move downstream except square wave.

Acknowledgment

The authors express their gratitude to TEQIP-II, KNIT, Sultanpur (U. P.), India for financial support during the course of the above study.

References

[1] Latham T. W. (1966) Fluid Motion in a Peristaltic Pump, M.S. Thesis, M.I.T. Cambridge, Massachusetts.
[2] Shapiro A. H., Jaffrin M. Y. and Weinberg S. L. (1969) "Peristaltic pumping with long wavelength at low Reynolds number", J. Fluid Mech, 37: 799-825.
[3] Victor S. A., and Shah V. L. (1975) "Heat Transfer To Blood Flowing In A Tube", Biorehology, 12: 361-68.
[4] Kothandapani M., and Srinivas S. (2008) "Peristaltic transport in an asymmetric channel with heat transfer a note", Int Commun Heat Mass Transfer, 35: 514-522.
[5] Radhakrishnamacharya G. and Murty V. R. (1993) "Heat Transfer to Peristaltic Transport in a Non Uniform Channel", Defence Science Journal, 43: 275-280.
[6] Hariharan P., Sheshadri V. and Banerjee R. K. (2008) "Peristaltic Transport of Non-Newtonian Fluid in diverging Tube with Different Wave Forms", Mathematical and Computer Modelling, 48: 998-1017.
[7] Nadeem S., and Akbar N. S.. (2009) "Influence of Heat Transfer on a Peristaltic Transport of Herchel-Bulkley Fluid in Non-Uniform Inclined Tube", Commun Nonlinear Sci Numer Simulat, 14: 4100-4113.
[8] Elllahi R., Bhatti M. M. and Vafai K. (2014) "Effects of Heat And Mass Transfer on Peristaltic Flow In a Non-Uniform Rectangular Duct", International Journal of Heat and Mass Transfer, 71: 706-19.
[9] Hayat T., Nadeem S. and Asghar S. (2004) "Periodic unidirectional flows of a viscoelastic fluid with the fractional Maxwell model", Appl. Math. Comput. 151 (1):153-161.
[10] Tripathi D., Pandey S. K. and Das S. (2010) "Peristaltic flow of viscoelastic fluid with fractional Maxwell model through a channel", Applied Mathematics and Computation, 215(10): 3645–3654.
[11] Tripathi D., Pandey S. K. and Das S. (2011) "Peristaltic transport of generalized burgers fluid: Application to the movement of chyme in small intestine", Acta astronautica, 69: 30-38.
[12] Tripathi D. (2011) "Peristaltic flow of fractional second grade fluid through a cylindrical tube", Thermal Science, 15(2): S167-S173.

[13] Narala V. K., Prasad K.M. and Ramanamurthy V.J. (2013) "Peristaltic Motion of Viscoelastic Fluid with Fractional second Grade Model in Curved Channels".Chinese Journal of Engineering,7.

[14] Rathod V. P. and Channakote M. M. (2014) "Interaction of heat transfer and peristaltic pumping of fractional second grade fluid through a vertical cylindrical tube", Thermal Science,18:1109-1118.

[15] Hameed M., Khan A. A., Ellahi R. and Raza M. (2015) "Study of magnetic and heat transfer on peristaltic transport of a fractional second grade fluid in vertical tube", Engineering Science and Technology an International Journal,18:496-510.