A Computational Analysis of a Two-Fluid non-Linear Mathematical Model of Pulsatile Blood Flow through Constricted Artery

S. U. Siddiqui, S. R. Shah, Geeta*

Department of Mathematics, Harcourt Butler Technological Institute, Kanpur-208002, India
*E-mail: geeta05_hbti@rediffmail.com

Abstract

This paper deals with the pulsatile flow of blood through a stenosed artery with the effect of body acceleration. We have considered the axially non-symmetric mild stenosis & a two layered blood flow with a core region of suspension of all erythrocytes assumed to be a Casson fluid and a peripheral layer of plasma free from cells as a Newtonian fluid. The non-linear differential equations governing the fluid flow are solved analytically and obtained the expressions for velocity, flow rate, wall shear stress, plug core radius, effective viscosity. We have discussed the effect of body acceleration, pulsatality, peripheral stenosis height and non-Newtonian behavior of blood on above mentioned flow quantities. It is found that the increase of stenosis size and yield stress increases the plug core radius, pressure drop, wall shear stress where as velocity and flow rate decreases. Body acceleration also plays a very important role in blood flow. The present study is more useful for the purpose of validation of the different models for blood flow in the different cases of stenosis.

Keywrds: Atherosclerosis; Wall shear stress; Axial velocity; Body acceleration;

Introduction

Atherosclerosis is a major risk factor for many conditions involving the flow of blood. It is a potentially serious condition where arteries becomes clogged up by fatty substances known as plaques or atheroma. The plaques cause affected arteries to harden and narrow, which can be dangerous as restricted blood flow can damage organs and stop them functioning properly. According to the experts stenosis begins with the damage to the endothelium caused by the high blood pressure, smoking or high cholesterol.

The flow of blood through a stenosed artery can be represented by different fluid models according to the situation. In the literature it is found that some researchers represented blood by Newtonian & non-Newtonian while some of the researchers considered single and two layered blood flow. Arteries are the blood vessels, carry blood from the heart throughout the body. Human heart pumps the blood in the circulatory system and produces the pressure gradient throughout the system. There are two components of the pressure gradient, one is constant and other is fluctuating or pulsatile.

Body acceleration is a very important factor in blood flow modeling. Body acceleration disturbs the normal blood flow and causes many problems such as headache, abdominal pain, increases pulse rate and others. Rathod et. al. [12] describes the pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and magnetic field. Shaw et. al. [15] have shown the effect of body acceleration on the two dimensional flow of casson fluid through an artery with asymmetric stenosis where, the artery wall has been treated as an elastic cylindrical tube.

Srivastava [20] modeled a two layered blood flow through a narrow catheterized artery by considering blood as Newtonian, incompressible with variable blood viscosity. Bugliarello and Sevilla [2] and Hayden [3] have experimentally observed that when blood flows through narrow tubes there exists a cell free plasma layer near the wall. In view of their experiments, it is preferable to represent the flow of blood through narrow tubes by a two layered model instead of single layered model. Pulsatile couple stress fluid model through stenosed artery with the porous effect for non-Newtonian blood has been considered by Singh and Rathee [17]. Singh and Singh [16] presented a paper that deals with the blood flow through a radially non-symmetric stenosed artery considering blood as a non-Newtonian casson fluid model.
Few authors focused on the pulsatile nature of blood, magnetic effect and body acceleration in stenotic artery with its application in different blood disease [1, 4, 8]. Mathematically the solution can be done by using Hankel transformation approach instead of either numerical or empirical approach by Haghighi [8]. Sankar et. al. [13] examine the effect of an external magnetic field on the blood flow through a composite stenosis by using a two layered blood flow model consisting of a cell free peripheral layer and a core region of erythrocytes in plasma flowing through a composite stenosis in the presence of an external transverse magnetic field.
Hazarika and Sharma [9] considered a two-layered mathematical model for blood flow through tapering asymmetric stenosed artery with velocity slip at the interface under the effect of transverse magnetic field. Srivastava and saxena [18] investigated two-layered model of casson's fluid flow through stenotic blood vessels. Ponalgusamy [11] investigated blood flow through an artery with mild stenosis: A two-layered model, different shapes of stenosis and slip at the wall.
Ellahi et. al. [7] considered a study of non-Newtonian micropolar fluid in an arterial blood flow through composite stenosis, slip velocity are taken into account with permeable wall effects.

In the present analysis an attempt has been made to provide a model to examine the effect of an body acceleration on blood flow through an axially symmetric stenosis which has not yet been examined in previous works. For this purpose we have used a two-layered blood flow model consisting of a cell-free peripheral layer and a core region of erythrocytes in plasma. This two-layered model for blood flow provides a more realistic model for flow in small arteries since we cannot neglect the existence of the peripheral layer and the red blood cells in the plasma. The effect that the body acceleration has on the fluid's velocity, flow rate, wall shear stress and shear stress at the stenosis throat will be examined. Present model can lead to the improvement of existing diagnostic tools for a more effective treatment of patients suffering from cancer, hypertension, myocardial infarction, stroke and paralysis.

Mathematical formulation

We consider an axially symmetric, laminar, pusatile and fully developed flow of blood (assumed to be incompressible) through a circular tube with an axilly symmetric mild stenosis as shown in figure. It is assumed that the wall of the tube is rigid and the body fluid blood is represented by a two-fluid model with a core region of suspension of all erythrocytes as a cosson fluid and a peripheral layer of plasma as a Newtonian fluid. The artery length is assumed to be large enough as compared to it's radius so that the entrance and the exit, special wall effects can be neglected.

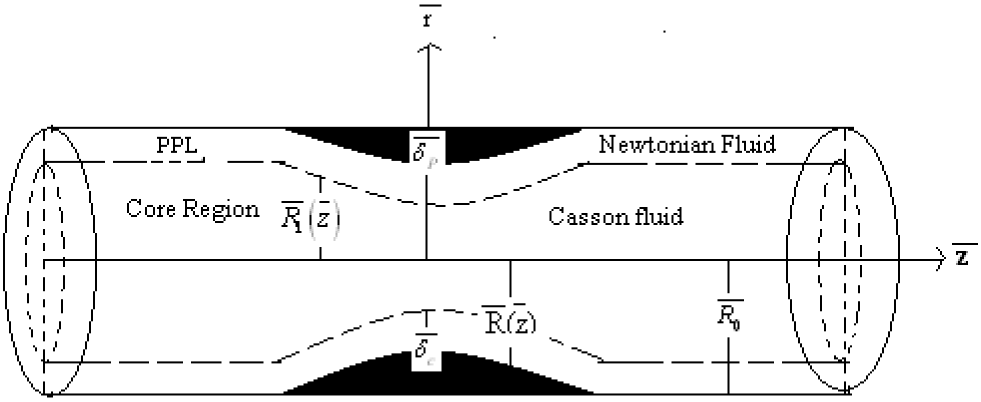

Figure 1. The geometry of an axially symmetric arterial stenosis

The geometry of the stenosis in the peripheral region is given by

$$\bar{R}\left(\bar{z}\right)=\begin{cases}\bar{R}_0-\left(\bar{\delta}_P/2\right)\left(1+\cos\left(\pi\bar{z}/\bar{z}_0\right)\right) & ,For\left|\bar{z}\right|\le\bar{z}_0\\ \bar{R}_0 & ,For\left|\bar{z}\right|>\bar{z}_0\end{cases}\tag{1}$$

The geometry of stenosis in the core region is given by

$$\bar{R}_1\left(\bar{z}\right)=\begin{cases}\beta\bar{R}_0-\left(\bar{\delta}_c/2\right)\left(1+\cos\left(\pi\bar{z}/\bar{z}_0\right)\right) & ,For\left|\bar{z}\right|\le\bar{z}_0\\ \beta\bar{R}_0 & ,For\left|\bar{z}\right|>\bar{z}_0\end{cases}\tag{2}$$

Where $\bar{R}\left(\bar{z}\right)$ is the radius of the stenosed artery with peripheral layer, $\bar{R}_1\left(\bar{z}\right)$ is the radius of the artery in the stenosed core region such that $\bar{R}_1\left(\bar{z}\right)=\beta\bar{R}(\bar{z})$, $\bar{R}_0$ and $\beta\bar{R}_0$ are the radii of the normal artery and core region of the normal artery respectively; $\bar{\delta}_P$ is the maximum height of the stenosis in the peripheral region, β is the ratio of the central core radius to the normal artery radius, $\bar{\delta}_c$ is the maximum height of the stenosis in the core region such that $\bar{\delta}_c=\beta\bar{\delta}_P$ and $\bar{z}_0$ is the half length of the stenosis. It has been reported that the radial velocity is the negligible small for a low Reynolds's number flow in a tube with mild stenosis.
The equation of motion governing the fluid flow are given by

$$\bar{\rho}_c\frac{\partial\bar{u}_c}{\partial\bar{t}}=-\frac{\partial\bar{p}}{\partial\bar{z}}-\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\bar{\tau}_c\right)+\bar{F}(\bar{t})\qquad 0\le\bar{r}\le\bar{R}_1(\bar{z})\tag{3}$$

$$\bar{\rho}_N\frac{\partial\bar{u}_N}{\partial\bar{t}}=-\frac{\partial\bar{p}}{\partial\bar{z}}-\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\bar{\tau}_N\right)+\bar{F}(\bar{t})\qquad \bar{R}_1(\bar{z})\le\bar{r}\le\bar{R}(\bar{z})\tag{4}$$

In the core and peripheral regions respectively, where $\bar{u}_c$ and $\bar{u}_N$ are the fluid velocities in the core region and peripheral regions respectively, $\bar{\tau}_c$ and $\bar{\tau}_N$ are the shear stresses

for the casson fluid and Newtonian fluid respectively, $\overline{\rho_c}$ and $\overline{\rho_N}$ are the densities for casson fluid and Newtonian fluid respectively, $\overline{P}$ is the pressure and $\overline{F}(\bar{t})$ is the body acceleration.
The consecutive equations for casson fluid and Newtonian fluid are respectively given by

$$\begin{cases} \left(\overline{\tau_c}\right)^{1/2} = \left(-\overline{\mu_c}\dfrac{\partial \overline{u_c}}{\partial \bar{r}}\right)^{1/2} + \left(\overline{\tau_y}\right)^{1/2} \quad if\ \overline{\tau_c} \ge \overline{\tau_y}\ and\ \overline{R_p} \le \bar{r} \le \overline{R_1}(\bar{z}) \\ \dfrac{\partial \overline{u_c}}{\partial \bar{r}} = 0\ if\ \overline{\tau_c} \ge \overline{\tau_y}\ and\ 0 \le \bar{r} \le \overline{R_p} \end{cases} \tag{5}$$

$$\overline{\tau_N} = -\overline{\mu_N}\frac{\partial \overline{u_N}}{\partial \bar{r}}\ if\ \overline{R_1}(\bar{z}) \le \bar{r} \le \overline{R}(\bar{z}) \tag{6}$$

Where $\overline{R_p}$ is the radius of the plug flow region.
The periodic body acceleration in the axial direction is given by

$$\overline{F}(\bar{t}) = a_0 \cos(\overline{\omega_b}\,\bar{t} + \phi) \tag{7}$$

Where a_0 is its amplitude, $\overline{\omega_b} = 2\pi \overline{f_b}$, $\overline{f_b}$ is its frequency in Hz., ϕ is the lead angle of $\overline{F}(\bar{t})$ with respected to the heart action. The frequency of body acceleration $\overline{f_b}$ is assumed to be small so that wave effect can be neglected.
The pressure gradient at any $\bar{z}$ and $\bar{t}$ may be represented as follows

$$\frac{\partial \bar{p}}{\partial \bar{z}}(\bar{z}, \bar{t}) = A_0 + A_1 \cos(\overline{\omega_p}\,\bar{t}) \tag{8}$$

Where A_0 the steady component of the pressure gradient is, A_1 is amplitude of the fluctuating component and $\overline{\omega_p} = 2\pi \overline{f_p}$ where $\overline{f_p}$ is the pulse frequency. Both A_0 and A_1 are function of $\bar{z}$ we introduce the following non-dimensional variables.

$$\begin{gathered} z = \frac{\bar{z}}{\overline{R_0}},\ R(z) = \frac{\overline{R(z)}}{\overline{R_0}},\ R_1(z) = \frac{\overline{R_1(z)}}{\overline{R_0}},\ r = \frac{\bar{r}}{\overline{R_0}},\ t = \bar{t}\,\overline{w_p},\ w = \frac{\overline{w_b}}{\overline{w_p}}, \\ \delta_p = \frac{\overline{\delta_p}}{\overline{R_0}},\ \delta_c = \frac{\overline{\delta_c}}{\overline{R_0}}, u_c = \frac{\overline{u_c}}{A_0\overline{R_0}^2 / 4\overline{r_c}},\ u_N = \frac{\overline{u_N}}{A_0\overline{R_0}^2 / 4\overline{r_N}}, \\ u_s = \frac{\overline{u_s}}{A_0\overline{R_0}^2 / 4\overline{r_N}},\ \tau_c = \frac{\overline{\tau_c}}{A_0\overline{R_0}/2},\ \tau_N = \frac{\overline{\tau_N}}{A_0\overline{R_0}/2}, e = \frac{A_1}{A_0},\ B = \frac{a_1}{A_0} \end{gathered} \tag{9}$$

Where α_c and α_N are pulsatile Reynolds's number for casson fluid and Newtonian fluid respectively.
Using non-dimensional variables, equation (1) and (2) becomes

$$R(z)=\begin{cases}1-(\delta_P/2)(1+\cos(\pi z/z_0)) & ,For\,|z|\le z_0\\ 1 & ,For\,|z|>z_0\end{cases} \tag{10}$$

$$R_1(z)=\begin{cases}\beta-(\delta_c/2)(1+\cos(\pi z/z_0)) & ,For\,|z|\le z_0\\ \beta & ,For\,|z|>z_0\end{cases} \tag{11}$$

The governing equation of motion given by equation (3) and (4) are represented in the non dimensional form as

$$\alpha_c^2\frac{\partial u_c}{\partial t}=4f(t)-\frac{2}{r}\frac{\partial}{\partial r}(r\tau_c) \tag{12}$$

$$\alpha_N^2\frac{\partial u_N}{\partial t}=4f(t)-\frac{2}{r}\frac{\partial}{\partial r}(r\tau_N) \tag{13}$$

where $f(t)=(1+e\cos t)+B\cos(\omega t+\phi)$ (14)

using non dimensional variables equation (5) & (6) reduce to

$$\tau_c^{1/2}=(-\frac{1}{2}\frac{\partial u_c}{\partial r})^{1/2}+\theta^{1/2} \tag{15}$$

$$\frac{\partial u_c}{\partial r}=0 \tag{16}$$

$$\tau_N=-\frac{1}{2}\frac{\partial u_N}{\partial r} \tag{17}$$

The boundary conditions are

$$\left.\begin{aligned}&\overline{\tau_c}\ \ is\ \ finite\ and\ \ \frac{\overline{\partial u_c}}{\partial \bar{r}}=0\ at\ \bar{r}=0\\ &\overline{u_N}=0\ at\ r=\overline{R}\\ &\overline{\tau_c}=\overline{\tau_N}\ \ and\ \ \overline{u_c}=\overline{u_N}\ \ at\ \bar{r}=\overline{R_1}\end{aligned}\right\} \tag{18}$$

The boundary conditions in the dimensionless form are

$$\left.\begin{aligned}&\tau_c\ \ is\ \ finite\ and\ \ \frac{\partial u_c}{\partial r}=0\ at\ r=0\\ &u_N=0\ at\ r=R\\ &\tau_c=\tau_N\ \ and\ \ u_c=u_N\ \ at\ r=R_1\end{aligned}\right\} \tag{19}$$

The non-dimensional volumetric flow rate is given by

$$Q = 4\int_0^{R(z)} u(z,r,t)rdr \tag{20}$$

Where $Q(t) = \dfrac{\bar{Q}(\bar{t})}{\pi\left(\bar{R}_0\right)^4 A_0 / 8\bar{\mu}_c}$; $\bar{Q}(\bar{t})$ is the volumetric flow rate.

Perturbation Method of Solution

Since it is not possible to find an exact solution to the system of nonlinear equations (12)-(17), the perturbation method is used to obtain the approximate solution to the unknowns u_c, u_N, τ_c and τ_N . when we non-dimensionalize the momentum equations (3) and (4) α_c^2 and α_N^2 occurs naturally and hence it is more appropriate to expand the equations (12)-(17) about α_c^2 and α_N^2.

Let us expand the plug core velocity u_p, the velocity in the core region u_c in the perturbation series of α_c^2 as below (where α_c^2 <<1)

$$u_p(z,t) = u_{0p}(z,t) + \alpha_c^2 u_{1p}(z,t) + ... \tag{21}$$

$$u_c(z,r,t) = u_{0c}(z,r,t) + \alpha_c^2 u_{1c}(z,r,t) + ... \tag{22}$$

$$R_p(z,t) = R_{0p}(z,t) + \alpha_c^2 R_{1p}(z,t) + ... \tag{23}$$

$$u_N(z,r,t) = u_{0N}(z,r,t) + \alpha_N^2 u_{1N}(z,r,t) + ... \tag{24}$$

Substituting the perturbation series expansion in equation (12), (15) and (16) and equating the power of α_c^2 and constant terms.

$$\frac{\partial u_{0c}}{\partial t} = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_{1c}) \tag{25}$$

$$2f(t)r = \frac{\partial}{\partial r}(r\tau_{0c}) \tag{26}$$

$$-\frac{\partial u_{0c}}{\partial r} = 2(\tau_{0c} - 2\sqrt{\theta\tau_{0c}} + \theta) \tag{27}$$

$$-\frac{\partial u_{1c}}{\partial r} = 2\tau_{1c}(1 - \sqrt{\theta/\tau_{0c}}) \tag{28}$$

Similarly using the perturbation series expansion in equation (13) and (17) then equating constant term and α_N^2 terms.

$$\left.\begin{aligned} &\frac{\partial}{\partial r}(r\tau_{0N}) = 2f(t)r && (a)\\ &\frac{\partial u_{0N}}{\partial t} = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_{1N}) && (b)\\ &-\frac{\partial u_{0N}}{\partial r} = 2\tau_{0N} && (c)\\ &-\frac{\partial u_{1N}}{\partial r} = 2\tau_{1N} && (d) \end{aligned}\right] \tag{29}$$

Now substituting the perturbation series expansion in equation (19) and then equating the constant term and α_c^2, α_N^2, we get

$$\left.\begin{aligned} &\tau_{0p}\ \ and\ \ \tau_{1p}\ \ are\ \ finite,\ \frac{\partial u_{0p}}{\partial r} = 0,\ \frac{\partial u_{1p}}{\partial r} = 0\ \ at\ \ r = 0\\ &\tau_{0c} = \tau_{0N},\ \tau_{1c} = \tau_{1N},\ u_{0c} = u_{0N},\ u_{1c} = u_{1N}\ \ at\ \ r = R_1\\ &u_{0N} = u_{1N} = 0\ at\ r = R\\ &\tau_{0c}\ \ and\ \ \tau_{1c}\ are\ finite\ at\ r = 0 \end{aligned}\right] \tag{30}$$

On solving equation (25)-(29) for unknowns $u_{0p}, u_{1p}, u_{0c}, u_{1c}, u_{0N}, u_{1N}, \tau_{0c}, \tau_{1c}, \tau_{0N}, \tau_{1N}$ using equation (30), we can obtain,

$$\tau_{0c} = f(t)r \tag{31}$$

$$\tau_{0N} = f(t)r \tag{32}$$

$$\tau_{0p} = f(t)R_{0p} \tag{33}$$

$$u_{0N} = f(t)R^2\left[1-\left(\frac{r}{R}\right)^2\right] \tag{34}$$

$$u_{0c} = f(t)(R^2 - r^2) + 2\theta(R_1 - r) - \frac{8}{3}\sqrt{\theta f(t)}\left(R_1^{\frac{3}{2}} - r^{\frac{3}{2}}\right) \tag{35}$$

$$u_{0p} = f(t)(R^2 - {r_{0p}}^2) + 2\theta(R_1 - r_{0p}) - \frac{8}{3}\sqrt{\theta f(t)}\left(R_1^{\frac{3}{2}} - {r_{0p}}^{\frac{3}{2}}\right) \tag{36}$$

$$\tau_{1c} = f'(t)\left[\frac{1}{8}(r^3 - 2R^2 r) + \frac{1}{21}k(7rR_1^{3/2} - 4r^{5/2})\right] \tag{37}$$

$$\tau_{1N} = -\frac{1}{8}f'(t)(2R^2 r - r^3) + \frac{k}{7r}f'(t)R_1^{7/2} \tag{38}$$

$$u_{1N} = \frac{f'(t)}{4}\left[R^2 r^2 - r^4 - \frac{3}{4}R^4 - \frac{8}{7}kR_1^{\frac{7}{2}}\ln\frac{r}{R}\right] \tag{39}$$

$$u_{1c} = f'(t)\begin{bmatrix} k\left(\dfrac{371}{2058}r^{\frac{7}{2}} - \dfrac{1}{3}R^2 r^{3/2} - \dfrac{1}{3}r^2 R_1^{3/2} + \dfrac{315}{2058}R_1^{\frac{7}{2}} + \dfrac{1}{3}R^2 R_1^{\frac{3}{2}}\right) \\ + \dfrac{4}{63}k^2(7r^{3/2}R_1^{3/2} - 2r^3 - 5R_1^3) + \\ \dfrac{1}{16}(R_1^4 - 4R^2R_1^2 - r^4 + 4R^2r^2) + \dfrac{1}{4}R^2R_1^2 - \dfrac{1}{4}R_1^4 - \dfrac{3}{16}R^4 - \dfrac{2}{7}kR_1^{7/2}\ln\dfrac{R_1}{R} \end{bmatrix} \quad (40)$$

$$u_{1p} = f'(t)\begin{bmatrix} k\left(\dfrac{371}{2058}R_{0p}^{\frac{7}{2}} - \dfrac{1}{3}R^2 R_{0p}^{\frac{3}{2}} - \dfrac{1}{3}R_{0p}^2 R_1^{3/2} + \dfrac{315}{2058}R_1^{\frac{7}{2}} + \dfrac{1}{3}R^2 R_1^{\frac{3}{2}}\right) \\ + \dfrac{4}{63}k^2(7R_{0p}^{\frac{3}{2}}R_1^{3/2} - 2R_{0p}^3 - 5R_1^3) + \\ \dfrac{1}{16}(R_1^4 - 4R^2R_1^2 - R_{0p}^4 + 4R^2R_{0p}^2) + \dfrac{1}{4}R^2R_1^2 - \dfrac{1}{4}R_1^4 - \dfrac{3}{16}R^4 - \dfrac{2}{7}kR_1^{7/2}\ln\dfrac{R_1}{R} \end{bmatrix} \quad (41)$$

Where $k^2 = \dfrac{\theta}{f(\mathrm{t})}$

Neglecting the terms of $o(\alpha_c^2)$ and higher power of α_c in equation (23), the first approximation plug core radius can be obtained as

$$R_{0p} = \theta / f(t) = k^2 \quad (42)$$

Using equations (34) (35), (39), & (40) the expressions for axial velocities in the core and peripheral regions are obtained as

$$u_N = f(t)(R^2 - r^2) + \alpha_N^2 \frac{f'(t)}{4}\left[R^2r^2 - r^4 - \frac{3}{4}R^4 - \frac{8}{7}kR_1^{\frac{7}{2}}\ln\frac{r}{R}\right] \quad (43)$$

On replacing the value of u_{0c} and u_{1c} in equation (44) we get the value of u_c

$$u_c = u_{0c} + \alpha_c^2 u_{1c} \quad (44)$$

The expression for wall shear stress τ_w can be obtained as

$$\tau_w = \left(\tau_{0N} + \alpha_N^2 \tau_{1N}\right)_{r=R}$$

$$\tau_w = f(t)R - f'(t)\alpha_N^2\left(\frac{R^3}{8} - \frac{k}{7R}R_1^{\frac{7}{2}}\right) \quad (45)$$

From equation (20), (43) and (44) the volumetric flow rate is given by

$$Q = 4\int_0^{R_{0p}} r\left(u_{0p} + \alpha_c^2 u_{1p}\right)dr + 4\int_{R_{0p}}^{R_1} r\left(u_{0c} + \alpha_c^2 u_{1c}\right)dr + 4\int_{R_1}^{R} r\left(u_{0N} + \alpha_N^2 u_{1N}\right)dr \quad (46)$$

4. Results and Discussions

In the present model an attempt has been made to evaluate some of the important characteristics of blood flow past an arterial stenosis with the effect of body acceleration and pulsatile pressure gradient. In order to point out the biological importance and to examining the validity of the model, computer codes are developed to evaluate the analytical solution for flow rate, velocity profile, wall shear stress, effective viscosity for different values of parameters involved in equations (31)-(46).

The value of womersley frequency parameters are taken as ($\alpha_B = \alpha_N = 0.5$). The body acceleration parameter is taken in the range 0-2, the pressure gradient parameter e is taken in the range 0.5, the ratio of the central core radius to the normal radius of the artery is taken as 0.95.we have considered the magnitude of the lead angle ϕ as 0.2, the range 0-0.5 is taken for the peripheral stenosis height, yield stress is taken as 0, 0.1.

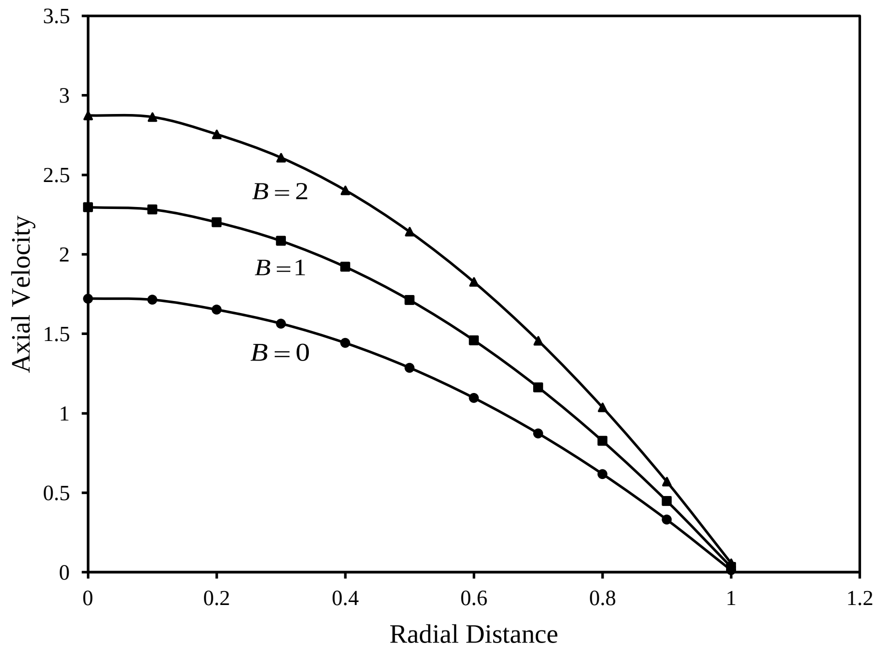

Fig. 2 Variation of axial velocity with radial distance
for $L_0 = 1, \delta_p = 0.2, \phi = 0.2, \beta = 0.8, e = 1$

Variation of axial velocity with radial distance has been shown in Fig. 2 and Fig. 3. From the Fig. 2 it is clearly observed that at $r = 0$ velocity is maximum and minimum value at the stenotic wall $r = R(z)$ for fixed values of $L_0 = 1, \delta_p = 0.2, t = 45^{o}, \beta = 0.8, e = 1$ and different values of body acceleration parameter $(B = 0,1,2)$ .It can be seen from Fig. 2 that axial velocity increases with the increase in body acceleration. Fig. 3 shows the variation of axial velocity with radial distance for different values of time t. Time plays a very important role in blood flow modeling. It is clearly seen from the Fig. 3, as time increases velocity decreases. When the blood is flowing to start i.e. when $t = 0$ velocity is maximum. Now as we increase the time, velocity decreases. Velocity is less at (t=1, 1.5) than velocity at $t = 0$.

Fig. 4 presents the flow rate distributions for the two fluid Casson models $R_{0p} = 0.1, \delta_p = 0.2, \beta = 0.8$ and $t = 45^{\circ}$ at the throat of the stenosis (i.e.) when $z = 0$. Like the axial velocity flow rate also increases with the increase in body acceleration. In the absence of yield stress, the curves are linear while the curves are nonlinear with the increase in yield stress θ. It is also observed that when the yield stress θ increases from θ=0 to θ=0.1, flow rate decreases because of increase in width of the plug flow region.

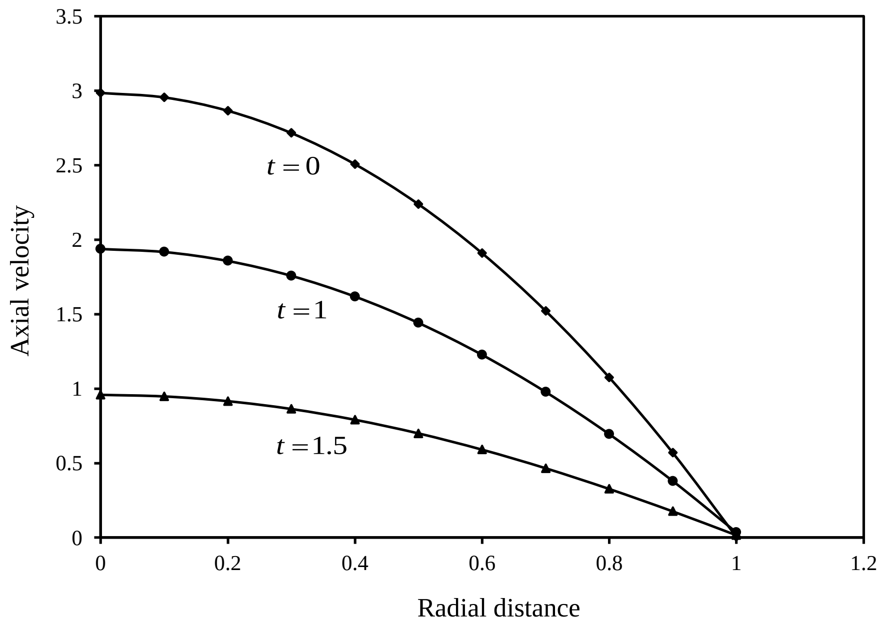

Fig. 3 Variation of axial velocity with the radial distance for different values of time and fixed values of $B = 1, \alpha_N = 0.5, \phi = 0.2, \theta = 0.1$

The variation of wall shear stress with time and peripheral stenosis height is described in figures (5,6 and 7) for different flow parameters ($\theta, e, B, \delta_p, \phi$). Wall shear stress is symmtrical about $t = 180^{\circ}$. It is observed that the wall shear stress increases linearly with the increase of the peripheral stenosis height from Fig. 5. Body acceleration also enhances the wall shear stress.

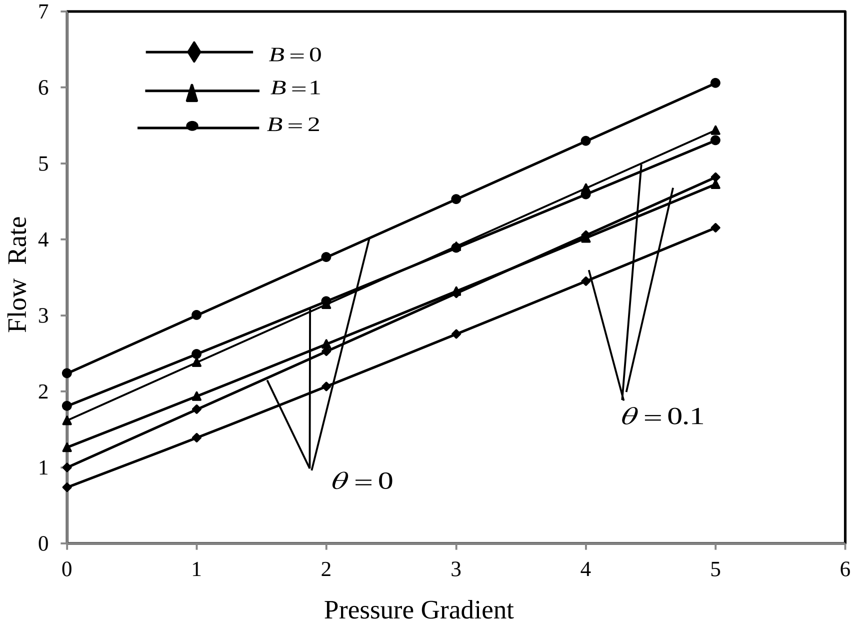

Fig 4 Variation of flow rate with pressure gradient e for $R_{0p}=0.1, \delta_p=0.2, \beta=0.8, t=45^{\circ}, z=0$

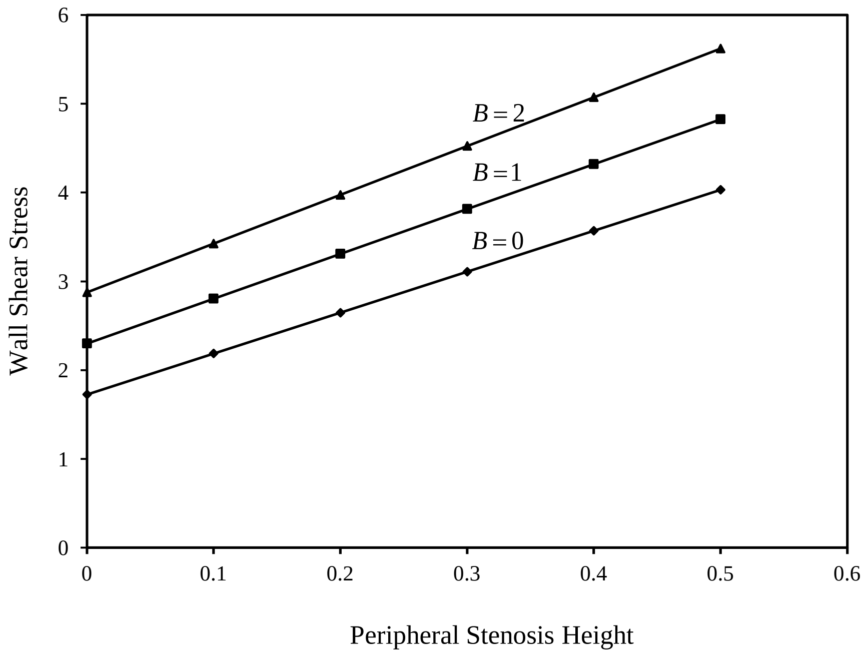

Fig. 5 Variation of wall shear stress with peripheral stenosis height for $B=1, \delta_c=0.5, \alpha_N=0.5, \beta=0.95, t=45^{\circ}$

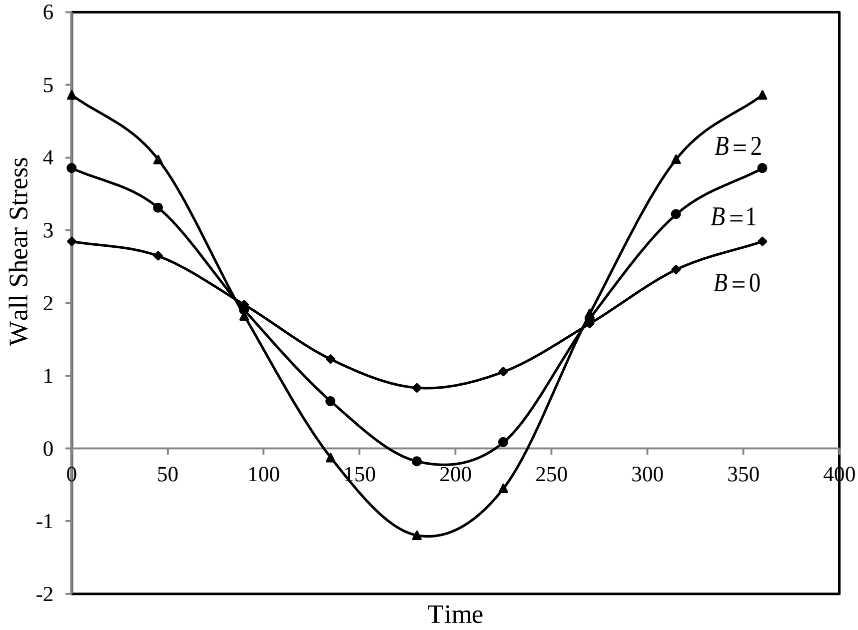

Fig. 6 Variation of wall shear stress with time t for
$\delta_c = 0.4, \delta_p = 0.2, \beta = 0.95, e = 1, \theta = 0.1$

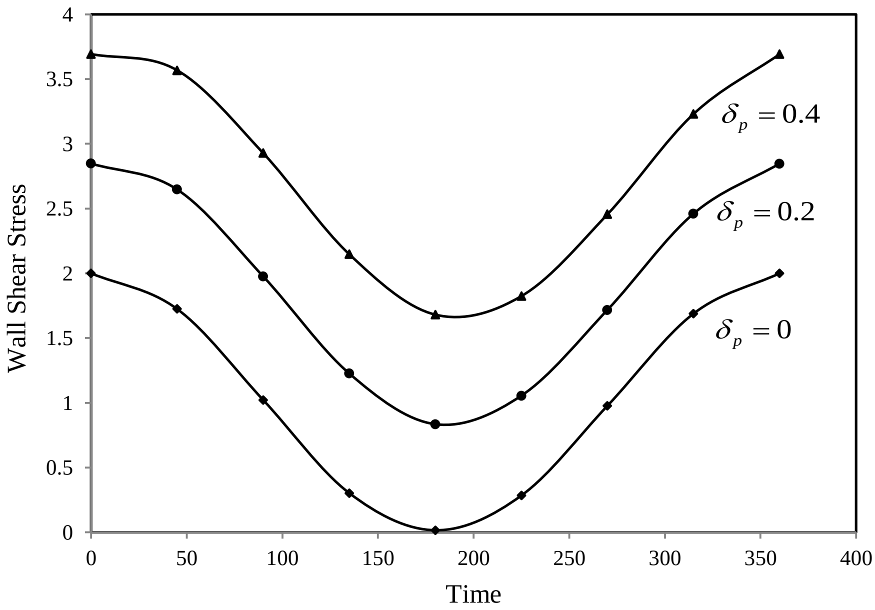

Fig. 7 Variation of Wall shear stress with time for
$\delta_c = 0.4, \beta = 0.95, e = 1, \theta = 0.1, \phi = 0.2$

Fig. 6 and Fig. 7 shows the variation of wall shear stress with time t, full scale of time $t(0^\circ \le t \le 360^\circ)$ has been considered. It is found that in a stenosed artery wall shear stress is highly influenced by body acceleration. Wall shear stress decreases with the increase in body acceleration. It is dipected that attains it's minimum value at $t = 180^\circ$ while, maximum at $t = 0^\circ$ and $t = 180^\circ$. We can not ignore the effect of peripheral stenosis height, since it is a very important factor in two layered blood flow modeling. Increase in peripheral stenosis height results an increase in wall shear stress.

Conclusion

We have considered the two-layered blood flow through stenosed artery with all erythrocytes in core region as a Casson fluid and peripheral layer of plasma as a Newtonian fluid. From the present results, it is clear that body acceleration is an important factor in blood flow modeling. In the present paper we have studied the effect of body acceleration on various flow parameters. It is depicted that flow rate and axial velocity increases but wall shear stress shows both increasing and decreasing trends with the increase in body acceleration parameter B, according to our imagination. This model is able to predict the some blood flow characteristics and may be useful in biomedical applications. In the near future the present study can be extended by considering magnetic effect of blood and permeability of wall.

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