

A Casson Fluid Model for multiple Stenosed Artery in the Presence of Magnetic Field

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Abstract

The flow of blood through a multi-stenosed artery under the influence of external applied magnetic field is studied. The artery is modeled as a circular tube. The effect of non-Newtonian nature of blood in small blood vessels has been taken into account by modeling blood as a Casson fluid. The effect of magnetic field, height of stenosis, parameter determining the shape of the stenosis on velocities, volumetric flow rate in stenotic region and wall shear stress at surface of stenosis are obtained and shown graphically. Some important observations regarding the flow of blood in multi stenosed artery are obtained leading to medical interest.

Key Words: Blood flow, magnetic effect, multiple stenosis artery, Casson fluid, shear stress, blood flow rate.

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Introduction

Stenosis narrowing of body passage tube [1] can cause series circulatory disorders by reducing the blood supply. Stenosis in the arteries supplying blood to the brain can cause cerebral strokes, and in coronary arteries, myocardial infarction, leading to the heart failure. The actual causes of the stenosis are not known, but it has been suggested that cholesterol deposition in arterial wall and proliferation of connective tissues may be responsible [2], vascular fluid dynamics is reported to play a significant role in the development and progression of the pathological conditions [3].

Blood is suspension of cells in plasma. Due to the presence of hemoglobin (an iron compound) in red cells, blood can be regarded as a suspension of magnetic particle (red cells) in non-magnetic plasma. The effect of a magnetic field on blood flow has been analyzed theoretically by treating blood as an electrically conductive fluid [4]. The conductive flow in the presence of a magnetic field induces voltage and currents, resulting in a decrease in the flow. The importance of heat transfer on artery diseases and blood flow was mentioned by several researcher. Ugulu and Abby [5] claimed that, the heat transfer and a magnetic field have a significant effect on blood flow through constricted artery.

An analytical solution for the steady flow of a viscous fluid through an arbitrary shaped tube of variable cross-section has been presented by Manton [6] using the ideas of steady lubrication theory Ramachandra Rao and Devanathan [7] and Hall [8] have extended the results of Manton [6] for unsteady pulsatile flows. The steady and unsteady flow through

channels and tubes of variable cross-section have been studied by Smith [9] and Duck [10]. Mathematical model for analyzing pulsatile flow in a single stenosed vessels have been proposed by Padmanabhan [11], Mehrotra et al. [12] and Mishra and Chakravorty [13].

The studies on the blood flow / unsteady blood flow through an artery with mild stenosis [14,15], effect of arterial dispensability on blood flow through model of mild axisymmetric arterial stenosis [16], flow of micro-polar fluid through a tube with a stenosis [17] non-Newtonian aspects of blood flow through stenosed arteries [18], flow of couple stress fluid through stenotic blood vessels [19], pulsatile flow of Casson's fluid through stenosed tube [20], oscillatory flow of blood in a stenosed artery [21] and in a single constricted blood vessels [22], effect of erythrocytes on blood flow characteristics in an indented tube [23], effect of an externally applied uniform magnetic field on the blood flow in a single constricted blood vessel [24] were also reported. In recent paper Manadal et al [25] developed a two dimension mathematical model to study the effect of externally imposed periodic body acceleration on non-Newtonian blood flow through an elastic stenosed artery where the blood is characterized by the generalized power-law model.

In all the above studies none has applied magnetic field. But the application of magneto hydrodynamics principles in medicine and biology is of growing interest in the literature of bio-mathematics [26, 27, 28]. By Lenz's law, the Lorentz's force will oppose the motion of conducting fluid. Since blood is an electrically conducting fluid, The MHD principles may be used to deaccelerate the flow of blood in a human arterial system and thereby it is useful in the treatment of certain cardiovascular disorders and in the diseases which accelerate blood circulation like hemorrhages and hypertension etc. [29]. It is well known that blood being a suspension of cells, behaves as a non-Newtonian fluid at low shear rates and during its flow through small blood vessels, especially in diseased states, when clotting effects in small arteries are present. Experimentally behavior of blood at low shear rates can best be described by Casson model.

Therefore, our aim to study the effect of an externally applied uniform magnetic field on the multi-stenosed artery. Blood is modeled as a Casson fluid by properly accounting for yield stress of blood in small blood vessel. The analytical expressions for the velocities (in normal and core region), blood flow rate and wall shear stress are obtained. The effect of external magnetic field and other parameter has been shown graphically in these results.

Mathematical formulation

Let us consider the Casson fluid motion of blood through a multi-stenosed artery under the influence of an external applied uniform transverse magnetic fluid. The geometry of the stenosis is as shown in fig.1. We have taken some assumption for solving the model.

- (1) Let us take the flow of blood as axially symmetric and fully developed (i.e. $v_r = v_\theta = 0$ flow in z-direction only). This is entirely reasonable and reinforces the fact that in steady –state incompressible flow in a circular tube of uniform cross-section. The velocity does not change in the direction of the flow, except near the entrance and exist regions.
- (2) Consider blood as a Casson fluid (non-Newtonian) and magnetic fluid. Since red cell is a major biomagnetic substance and blood flow may be influenced by the magnetic field.
- (3) Consider the transverse magnetic field. Since the biomagnetic fluid (blood) is subjected to a magnetic field, the action of magnetization will introduce a rotational motion to orient the magnetic fluid particle with the magnetic field).

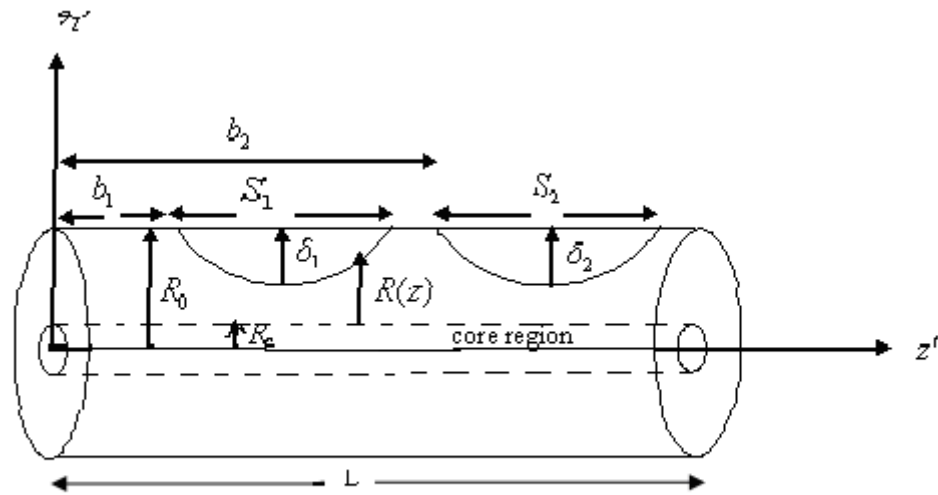


Fig.1: Geometry of multiple stenosed artery

The above assumptions for Navier-Stokes equation is given by,

$$-\frac{\partial P'}{\partial z'} + \frac{1}{r'} \frac{\partial}{\partial r'} (r' \tau'_c) + \mu_0 M \frac{\partial H'}{\partial z'} = 0 \quad (1)$$

Where r' and z' denote the radial and axial coordinates respectively, μ_0 magnetic permeability, M magnetization, H' magnetic field intensity, P' pressure and τ'_c the shear stress. For Casson fluid the relation between shear stress and shear rate is given by Fung [30],

$$\begin{aligned} \tau'_c{}^{1/2} &= \tau'_0{}^{1/2} + \left[\mu \left(-\frac{\partial u'}{\partial r'} \right) \right]^{1/2} & , \text{if } \tau'_c \geq \tau'_0 \\ \frac{\partial u'}{\partial r'} &= 0 & , \text{if } \tau'_c \leq \tau'_0 \end{aligned} \quad (2)$$

Where τ'_0 denotes yields stress and μ the viscosity of blood.

The boundary conditions appropriate to the problem under study are

$$(i) \quad u' = 0 \quad \text{at} \quad r' = R'(z) \quad (3a)$$

$$(ii) \quad \tau'_c \text{ is finite at } r' = 0 \quad (3b)$$

$$(iii) \quad \text{In core region } u' = u'_c \quad \text{at} \quad r' = R'_c \quad (3c)$$

Here u'_c is core velocity.

Solution of the problem: Introducing the following non-dimensional scheme.

$$\left. \begin{aligned} r &= \frac{r'}{R_0}, \quad z = \frac{z'}{R_0}, \quad R = \frac{R'}{R_0}, \quad P = \frac{P'}{\rho U_0^2} \\ u &= \frac{u'}{U_0}, \quad \delta = \frac{\delta'}{R_0}, \quad \tau = \frac{\tau'}{\rho U_0^2}, \quad H = \frac{H'}{H_0} \end{aligned} \right\} \quad (4)$$

Where H_0 is external transverse uniform constant magnetic field.

Using the non-dimensional scheme the governing equations from (1)-(3) are written as:

$$-\frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r\tau_c) + F_1 \frac{\partial H}{\partial z} = 0 \quad (5)$$

$$\tau_c^{1/2} = \tau_0^{1/2} + F_2 \left(-\frac{\partial u}{\partial r} \right)^{1/2}, \text{ if } \tau_c \geq \tau_0$$

$$\frac{\partial u}{\partial r} = 0, \text{ if } \tau_c \leq \tau_0$$

Where $F_1 = \frac{\mu_0 M H_0}{\rho U_0^2}$, $F_2 = \sqrt{\frac{\mu}{R_0 \rho U_0}}$

The boundary conditions (3a, 3b, 3c) reduce to

(i) $u = 0$ at $r = R(z)$ (7a)

(ii) τ_c is finite at $r = 0$ (7b)

(iii) In core region $u = u_c$ at $r = R_c$ (7c)

The geometry of the stenosis in non-dimensional form is given as

$$R(z) = 1 - c \left[S_L^{\Gamma-1} \{z - (b_1 + b_2)\} - \{z - (b_1 + b_2)\}^\Gamma \right],$$

$$= 1, \text{ otherwise}$$

$$(b_1 + b_2) \leq z \leq (b_1 + b_2) + S_L$$
(8)

Where $c = \frac{\delta}{R_0 (S_L)^\Gamma}$, (9)

δ is maximum height of stenosis

$$z = b_1 + b_2 + \frac{S_L}{\Gamma^{\Gamma-1}}$$
(10)

Where $S (\geq 2)$ is the parameter for determining the shape of the stenosis. $\frac{\delta}{R_0} \ll 1$.

Solution: On using analytical method in equations (5), (7) and using boundary conditions (7a, 7b, 7c) and (8) the expression for velocity u and core velocity u_c are:

$$u = \frac{1}{3F_2^2} \sqrt{2\tau_0} \left\{ \frac{\partial P}{\partial z} - F_1 \frac{dH}{dz} \right\}^{1/2} \left\{ r^{3/2} - R^{3/2}(z) \right\} - \frac{\tau_0}{2F_2^2} \{r - R(z)\} - \frac{1}{4F_2^2} \left\{ \frac{\partial P}{\partial z} - F_1 \frac{dH}{dz} \right\} \{r^2 - R^2(z)\}$$
(11)

$$u_c = \frac{1}{3F_2^2} \sqrt{2\tau_0} \left\{ \frac{\partial P}{\partial z} - F_1 \frac{dH}{dz} \right\}^{\frac{1}{2}} \left\{ R_c^{\frac{3}{2}} - R^{\frac{3}{2}}(z) \right\} - \frac{\tau_0}{2F_2^2} \tau_0 \{ R_c - R(z) \} - \frac{1}{4F_2^2} \left\{ \frac{\partial P}{\partial z} - F_1 \frac{dH}{dz} \right\} \{ R_c^2 - R^2(z) \} \quad (12)$$

The volumetric flow rate Q is given by:

$$Q = 2\pi \int_0^{R_c} r u_c dr + 2\pi \int_{R_c}^{R(z)} r u dr = Q_c + Q_0 \quad (13)$$

Where Q_c and Q_0 are the flow rate in core and annular region of stenotic tube. Using the equations (11) and (12) in equation (13) then, flow rate Q is:

$$Q = \frac{\pi}{F_2^2} \left[\frac{1}{7} \sqrt{2\tau_0} \left(\frac{\partial P}{\partial z} - F_1 \frac{dH}{dz} \right)^{\frac{1}{2}} \left(R_c^{7/2} - R^{7/2}(z) \right) - \frac{1}{6} \tau_0 \left(R_c^3 - R^3(z) \right) - \frac{1}{8} \left(\frac{\partial P}{\partial z} - F_1 \frac{dH}{dz} \right) \left(R_c^4 - R^4(z) \right) \right] \quad (14)$$

The wall shear stress τ_w is defined as:

$$\tau_w = \mu \left(\frac{du}{dr} \right)_{r=R(z)} \quad (15)$$

On differentiating eqn. (11) with respect to r and substituting in eqn. (14), then τ_w is given by:

$$\tau_w = \frac{\mu}{2F_2^2} \left[\sqrt{2\tau_0} \left(\frac{\partial P}{\partial z} - F_1 \frac{dH}{dz} \right)^{\frac{1}{2}} R^{\frac{1}{2}}(z) - \tau_0 - \left(\frac{\partial P}{\partial z} - F_1 \frac{dH}{dz} \right) R(z) \right] \quad (16)$$

Results and Discussions

Fig.2 (a) shows the axial velocity (u) with radial axis (r) for different values of induced magnetic field gradient ($H = dH/dz$), when the magnetic field gradient ($H = dH/dz$) increases then the curve shifts towards the origin. This is due to the fact that as magnetic field applied on the body, the Laurentz force oppose the flow of blood and hence reduces its velocity. This result compare with Das [29] and Ponalagusamy [33].

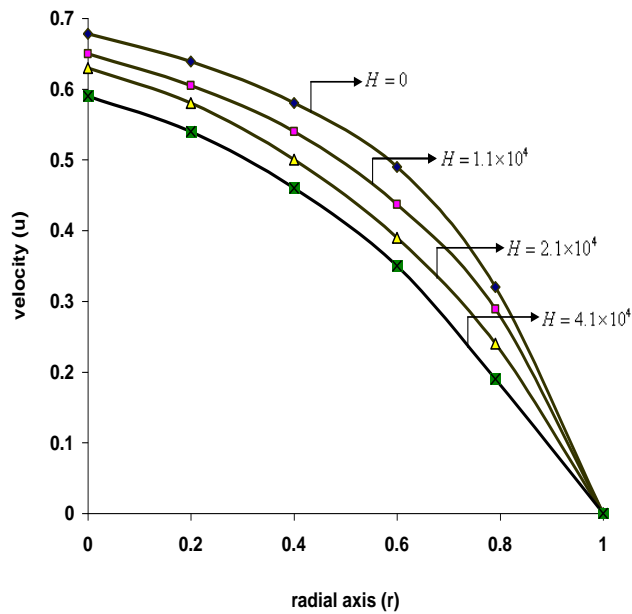


Fig.2 (a) :variation of velocity (u) with radial axis (r) for different value of magnetic field (H)

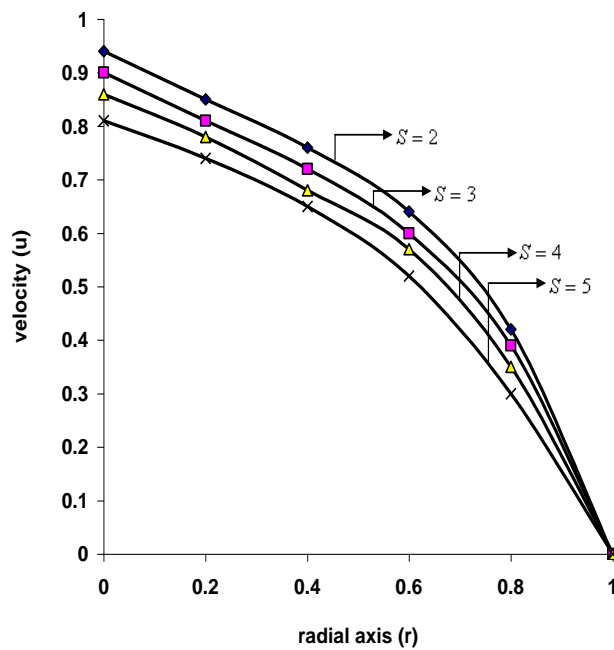


Fig.2 (b) :variation of velocity (u) with radial axis (r) for different value of the shape of the constriction (S)

Fig.2 (b) shows the axial velocity (u) with radial axis (r) for different values of shape of the constriction (S). The higher values of shape of the constriction (S) reduce the axial velocity in a large extent. It is also clear that the streaming blood is much higher than the non-Newtonian values. Thus the non-Newtonian characteristics of the flowing

blood affect the axial velocity profile. This result agrees qualitatively with Nagarani and Sarojamma [34].

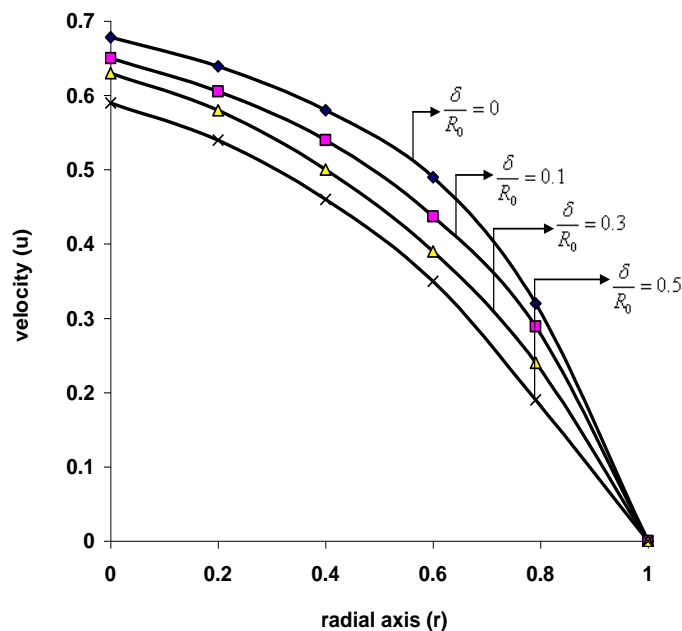


Fig.2 (c) :variation of velocity (u) with radial axis (r) for different values of stenosis height (δ/R_0)

The variation of velocity (u) with radial axis (r) for different values of ratio of the maximum height of stenosis and radius of the normal tube (δ/R_0) is shown in fig (2,c). Velocity decreases and it approaches zero when the ratio of the maximum height of stenosis and radius of the normal tube (δ/R_0) increased. Where high shearing velocity produced in order to attain uniform flow rate at given parameter, so the severity of the multi-stenosis affects the axial flow distribution significantly.

Figure 3, illustrate the variation of core (plug) velocity (u_c) with ratio of the stenosis height and radius of the normal tube (δ/R_0) for different values of induced magnetic gradient ($H = dH/dz$). The curves are all featured to be analogous in the sense that they do drop to zero on the wall surface from their maximum stenosis height ($\delta/R_0 = 0.5$). The core velocity decreases with increasing the magnetic gradient ($H = dH/dz$). This observation is in good agreement with those of Tzirtzilakis [35] although his studies were based on the Newtonian blood flow under the action of an applied magnetic field.

The variation of the flow rate (Q) with radial axis (r) of different values of magnetic gradient ($H = dH/dz$) and ratio of the stenosis height and radius of the normal tube (δ/R_0) is shown in fig .4 (a) and fig. 4 (b).

From fig. 4 (a), it is clear that the ratio of the stenosis height and radius of the normal tube (δ/R_0) increases the rate of flow diminishes appreciably for radial axis (r). The characterization of blood irrespective of the presence and absence of the magnetic field certainly ensures the importance of blood rheology in the flow phenomena. The flow rate diminishes as the artery gets narrowed gradually. It may be noted further that the

flow rate drops sharply with increasing severity of the constriction in the absence of the magnetic field.

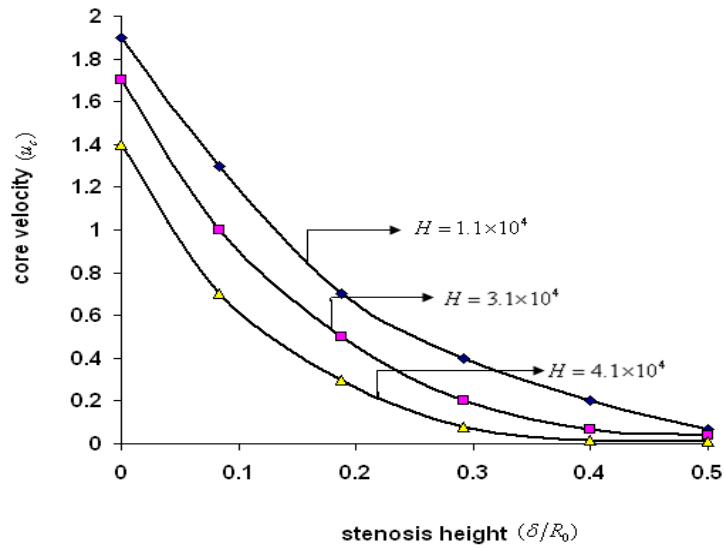


Fig.3:variation of core velocity(u_c) with stenosis height (δ/R_0) for different values of magnetic field(H)

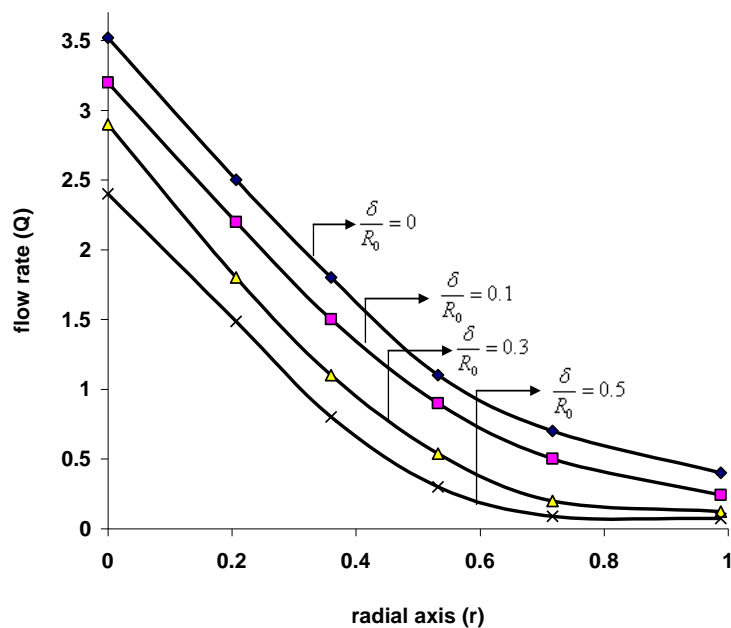


Fig.4 (a) :variation of flow rate (Q) with radial axis (r) for differnt value of stenosis height (δ/R_0)

In fig.4 (b), it is observed from the figure that in the presence of magnetic field gradient ($H = dH/dz$) the rate of blood flow increases at $r = 0$ and then diminishes for become the value of ($0 < r < 1$). The flow rate becomes higher in the absence of magnetic field and it gradually diminishes with increasing magnetic field gradient ($H = dH/dz$) which is in good agreement with these of Haik et al [36].

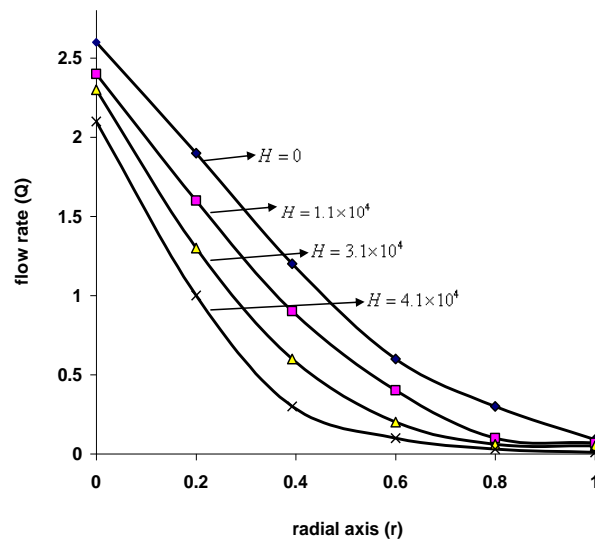


Fig.4 (b) :variation of flow rate (Q) with radial axis (r) for different values of magnetic field (H)

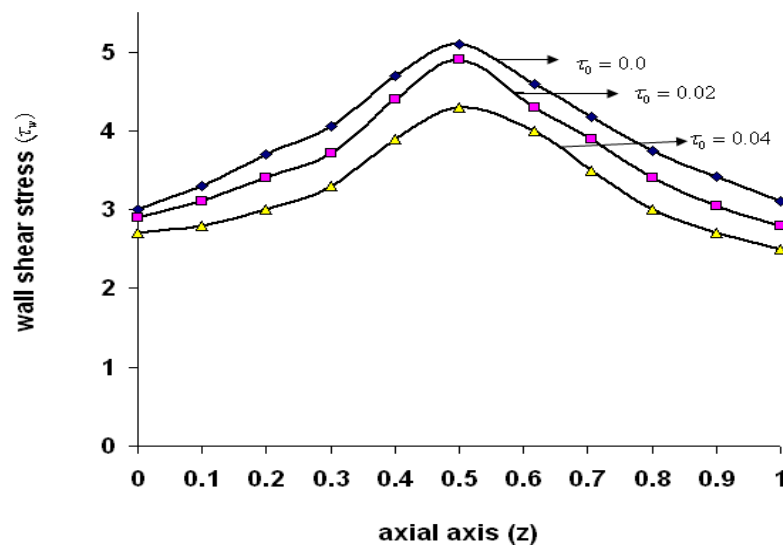


Fig.5 (a): variation of wall shear stress (τ_w) with axial axis (z) for different values of yield stress (τ_0)

Fig. 5(a) ,shows the result of the variation of wall shear stress (τ_w) with axial axis (z) for different values of yield stress (τ_0). It is noted that the wall shear stress increases as the axial distance z increases from (0 to 0.5) and then it decreases as z increases from (0.5 to 1). The maximum wall shear stress occurs at the middle of the stenosis. The wall shear stress decreases when the yield stress (τ_0) increases. The feature of these results is in good agreement with that of Srivastava and Saxena [37], whose studies were based on a one-dimensional Casson model of the blood flow in rigid arteries under steady-state conditions in the absence of magnetic field.

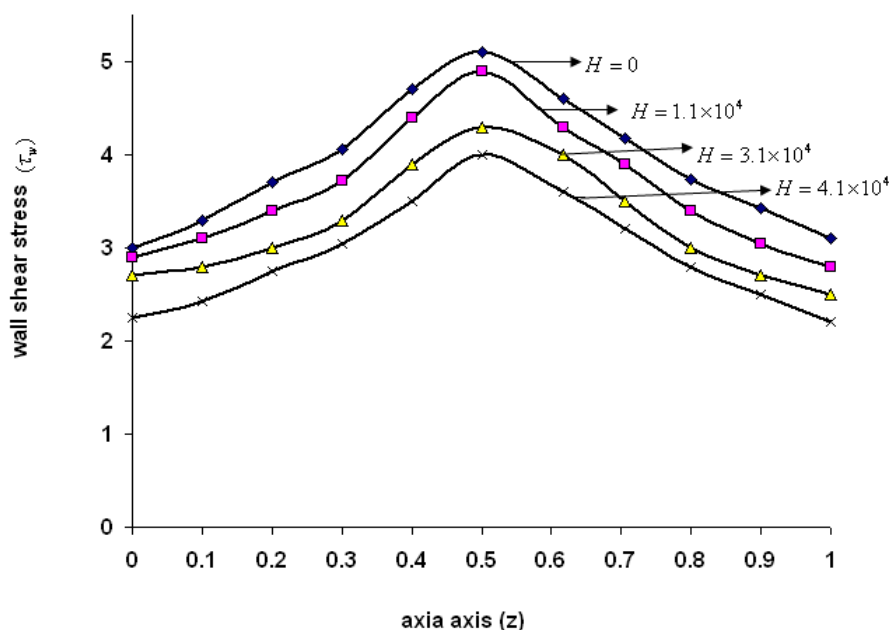


Fig.5 (b):variation of wall shear stress (τ_w) with axial axis(z) for different values of magnetic field (H)

Fig. 5(b), show the variation of wall shear stress (τ_w) with axial axis (z) for different magnetic field gradient ($H=dH/dz$).The wall shear stress increases rapidly as the length of arterial increases z from (0 to 0.5) and gradually decreases for the length of arterial increases z from (0.5 to 1).This is due to large velocity gradient and therefore the severity of the stenosis significantly affects the wall shear stress characteristics. It is also clear from the figure that when magnetic gradient ($H=dH/dz$). Increases then the wall shear stress decreases. However as the characteristic of the non-Newtonian fluid changes from shear-thinning to Newtonian. The wall shear stress is observed when the flowing blood is subjected to externally applied transverse magnetic field. The present stress distribution plays an important role in detecting the aggregation sites of platelets as mentioned by Fry [38].

Conclusions

From the above discussion, it is clear that the magnetic field, ratio of maximum height of stenosis and radius of normal tube and yield stress of the fluid are the strong parameters influencing the flow. It is observed that, in the presence of magnetic field the magnitude of velocity is decreased. The effect of yield stress and stenosis is to reduce the wall shear stress and flow rate in the presence of magnetic field. In view of these arguments, the present study may be more useful to control the blood flow in diseased state.

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